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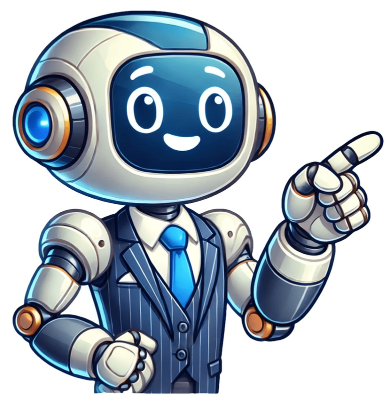

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The license may not give you all of the permissions necessary for your intended use. For example, other rights such as publicity, privacy, or moral rights may limit how you use the material. Random process independent of past historyA diagram representing a two-state Markov process. The numbers are the probability of changing from one state to another state.Part of a series on statisticsProbability theoryProbabilityAxiomsDeterminismSystemIndeterminismRandomnessProbability spaceSample spaceEventCollectively exhaustive eventsElementary eventMutual exclusivityOutcomeSingletonExperimentBernoulli trialProbability distributionBernoulli distributionBinomial distributionExponential distributionNormal distributionPareto distributionPoisson distributionProbability measureRandom variableBernoulli processContinuous or discreteExpected valueVarianceMarkov chainObserved valueRandom walkStochastic processComplementary eventJoint probabilityMarginal probabilityConditional probabilityIndependenceConditional independenceLaw of total probabilityLaw of large numbersBayes' theoremBoole's inequalityVenn diagramTree diagramvteIn probability theory and statistics, a Markov chain or Markov process is a stochastic process describing a sequence of possible events in which the probability of each event depends only on the state attained in the previous event. Informally, this may be thought of as, "What happens next depends only on the state of affairs now." A countably infinite sequence, in which the chain moves state at discrete time steps, gives a discrete-time Markov chain (DTMC). A continuous-time process is called a continuous-time Markov chain (CTMC). Markov processes are named in honor of the Russian mathematician Andrey Markov.Markov chains have many applications as statistical models of real-world processes.[1] They provide the basis for general stochastic simulation methods known as Markov chain Monte Carlo, which are used for simulating sampling from complex probability distributions, and have found application in areas including Bayesian statistics, biology, chemistry, economics, finance, information theory, physics, signal processing, and speech processing.[1][2][3]The adjectives Markovian and Markov are used to describe something that is related to a Markov process.[4]Russian mathematician Andrey MarkovA Markov process is a stochastic process that satisfies the Markov property (sometimes characterized as "memorylessness"). In simpler terms, it is a process for which predictions can be made regarding future outcomes based solely on its present state andmost importantlysuch predictions are just as good as the ones that could be made knowing the process's full history.[5] In other words, conditional on the present state of the system, its future and past states are independent.A Markov chain is a type of Markov process that has either a discrete state space or a discrete index set (often representing time), but the precise definition of a Markov chain varies.[6] For example, it is common to define a Markov chain as a Markov process in either discrete or continuous time with a countable state space (thus regardless of the nature of time),[7][8][9][10] but it is also common to define a Markov chain as having discrete time in either countable or continuous state space (thus regardless of the state space).[6]The system's state space and time parameter index need to be specified. The following table gives an overview of the different instances of Markov processes for different levels of state space generality and for discrete time v. continuous time:Countable state spaceContinuous or general state spaceDiscrete-time(discrete-time) Markov chain on a countable or finite state spaceMarkov chain on a measurable state space (for example, Harris chain)Continuous-timeContinuous-time Markov process or Markov jump processAny continuous stochastic process with the Markov property (for example, the Wiener process)Note that there is no definitive agreement in the literature on the use of some of the terms that signify special cases of Markov processes. Usually the term "Markov chain" is reserved for a process with a discrete set of times, that is, a discrete-time Markov chain (DTMC),[11] but a few authors use the term "Markov process" to refer to a continuous-time Markov chain (CTMC) without explicit mention.[12][13][14] In addition, there are other extensions of Markov processes that are referred to as such but do not necessarily fall within any of these four categories (see Markov model). Moreover, the time index need not necessarily be real-valued; like with the state space, there are conceivable processes that move through index sets with other mathematical constructs. Notice that the general state space continuous-time Markov chain is general to such a degree that it has no designated term.While the time parameter is usually discrete, the state space of a Markov chain does not have any generally agreed-on restrictions: the term may refer to a process on an arbitrary state space.[15] However, many applications of Markov chains employ finite or countably infinite state spaces, which have a more straightforward statistical analysis. Besides time-index and state-space parameters, there are many other variations, extensions and generalizations (see Variations). For simplicity, most of this article concentrates on the discrete-time, discrete state-space case, unless mentioned otherwise.The changes of state of the system are called transitions. The probabilities associated with various state changes are called transition probabilities. The process is characterized by a state space, a transition matrix describing the probabilities of particular transitions, and an initial state (or initial distribution) across the state space. By convention, we assume all possible states and transitions have been included in the definition of the process, so there is always a next state, and the process does not terminate.A discrete-time random process involves a system which is in a certain state at each step, with the state changing randomly between steps. The steps are often thought of as moments in time, but they can equally well refer to physical distance or any other discrete measurement. Formally, the steps are the integers or natural numbers, and the random process is a mapping of these to states. The Markov property states that the conditional probability distribution for the system at the next step (and in fact at all future steps) depends only on the current state of the system, and not additionally on the state of the system at previous steps.Since the system changes randomly, it is generally impossible to predict with certainty the state of a Markov chain at a given point in the future. However, the statistical properties of the system's future can be predicted. In many applications, it is these statistical properties that are important.Andrey Markov studied Markov processes in the early 20th century, publishing his first paper on the topic in 1906.[16][17][18] Markov Processes in continuous time were discovered long before his work in the early 20th century in the form of the Poisson process.[19][20][21] Markov was interested in studying an extension of independent random sequences, motivated by a disagreement with Pavel Nekrasov who claimed independence was necessary for the weak law of large numbers to hold.[22] In his first paper on Markov chains, published in 1906, Markov showed that under certain conditions the average outcomes of the Markov chain would converge to a fixed vector of values, so proving a weak law of large numbers without the independence assumption,[16][17][18] which had been commonly regarded as a requirement for such mathematical laws to hold.[18] Markov later used Markov chains to study the distribution of vowels in Eugene Onegin, written by Alexander Pushkin, and proved a central limit theorem for such chains.[16]In 1912 Henri Poincar studied Markov chains on finite groups with an aim to study card shuffling. Other early uses of Markov chains include a diffusion model, introduced by Paul and Tatyana Ehrenfest in 1907, and a branching process, introduced by Francis Galton and Henry William Watson in 1873, preceding the work of Markov.[16][17] After the work of Galton and Watson, it was later revealed that their branching process had been independently discovered and studied around three decades earlier by Irne-Jules Bienaym.[23] Starting in 1928, Maurice Frchet became interested in Markov chains, eventually resulting in him publishing in 1938 a detailed study on Markov chains.[16][24]Andrey Kolmogorov developed in a 1931 paper a large part of the early theory of continuous-time Markov processes.[25][26] Kolmogorov was partly inspired by Louis Bachelier's 1900 work on fluctuations in the stock market as well as Norbert Wiener's work on Einstein's model of Brownian movement.[25][27] He introduced and studied a particular set of Markov processes known as diffusion processes, where he derived a set of differential equations describing the processes.[25][28] Independent of Kolmogorov's work, Sydney Chapman derived in a 1928 paper an equation, now called the ChapmanKolmogorov equation, in a less mathematically rigorous way than Kolmogorov, while studying Brownian movement.[29] The differential equations are now called the Kolmogorov equations[30] or the KolmogorovChapman equations.[31] Other mathematicians who contributed significantly to the foundations of Markov processes include William Feller, starting in 1930s, and then later Eugene Dynkin, starting in the 1950s.[26]Main article: Examples of Markov chainsMark V. Shaney is a third-order Markov chain program, and a Markov text generator. It ingests the sample text (the Tao Te Ching, or the posts of a Usenet group) and creates a massive list of every sequence of three successive words (triplet) which occurs in the text. It then chooses two words at random, and looks for a word which follows those two in one of the triplets in its massive list. If there is more than one, it picks at random (identical triplets count separately, so a sequence which occurs twice is twice as likely to be picked as one which only occurs once). It then adds that word to the generated text. Then, in the same way, it picks a triplet that starts with the second and third words in the generated text, and that gives a fourth word. It adds the fourth word, then repeats with the third and fourth words, and so on.[32]Random walks based on integers and the gambler's ruin problem are examples of Markov processes.[33][34] Some variations of these processes were studied hundreds of years earlier in the context of independent variables.[35][36] Two important examples of Markov processes are the Wiener process, also known as the Brownian motion process, and the Poisson process,[19] which are considered the most important and central stochastic processes in the theory of stochastic processes.[37][38][39] These two processes are Markov processes in continuous time, while random walks on the integers and the gambler's ruin problem are examples of Markov processes in discrete time.[33][34]A famous Markov chain is the so-called "drunkard's walk", a random walk on the number line where, at each step, the position may change by +1 or 1 with equal probability. From any position there are two possible transitions, to the next or previous integer. The transition probabilities depend only on the current position, not on the manner in which the position was reached. For example, the transition probabilities from 5 to 4 and 5 to 6 are both 0.5, and all other transition probabilities from 5 are 0. These probabilities are independent of whether the system was previously in 4 or 6.A series of independent states (for example, a series of coin flips) satisfies the formal definition of a Markov chain. However, the theory is usually applied only when the probability distribution of the next state depends on the current one.Suppose that there is a coin purse containing five coins worth 25, five coins worth 10 and five coins worth 5, and one by one, coins are randomly drawn from the purse and are set on a table. If X n {\displaystyle X_{n}} represents the total value of the coins set on the table after n draws, with X 0 = 0 {\displaystyle X_{0}=0} , then the sequence { X n : n N } {\displaystyle \{X_{n}:n\in \mathbb {N} \}} is not a Markov process. To see why this is the case, suppose that in the first six draws, all five nickels and a quarter are drawn. Thus X 6 = $ 0.50 {\displaystyle X_{6}=\$0.50} . If we know not just X 6 {\displaystyle X_{6}} , but the earlier values as well, then we can determine which coins have been drawn, and we know that the next coin will not be a nickel; so we can determine that X 7 $ 0.60 {\displaystyle X_{7}\geq \$0.60} with probability 1. But if we do not know the earlier values, then based only on the value X 6 {\displaystyle X_{6}} we might guess that we had drawn four dimes and two nickels, in which case it would certainly be possible to draw another nickel next. Thus, our guesses about X 7 {\displaystyle X_{7}} are impacted by our knowledge of values prior to X 6 {\displaystyle X_{6}} .However, it is possible to model this scenario as a Markov process. Instead of defining X n {\displaystyle X_{n}} to represent the total value of the coins on the table, we could define X n {\displaystyle X_{n}} to represent the count of the various coin types on the table. For instance, X 6 = 1 , 0 , 5 {\displaystyle X_{6}=1,0,5} could be defined to represent the state where there is one quarter, zero dimes, and five nickels on the table after 6 one-by-one draws. This new model could be represented by 6 6 6 = 216 {\displaystyle 6\times 6\times 6=216} possible states, where each state represents the number of coins of each type (from 0 to 5) that are on the table. (Not all of these states are reachable within 6 draws.) Suppose that the first draw results in state X 1 = 0 , 1 , 0 {\displaystyle X_{1}=0,1,0} . The probability of achieving X 2 {\displaystyle X_{2}} now depends on X 1 {\displaystyle X_{1}} ; for example, the state X 2 = 1 , 0 , 1 {\displaystyle X_{2}=1,0,1} is not possible. After the second draw, the third draw depends on which coins have so far been drawn, but no longer only on the coins that were drawn for the first state (since probabilistically important information has since been added to the scenario). In this way, the likelihood of the X n = i , j , k {\displaystyle X_{n}=i,j,k} state depends exclusively on the outcome of the X n 1 = , m , p {\displaystyle X_{n-1}=\ell ,m,p} state.Main article: Discrete-time Markov chainA discrete-time Markov chain is a sequence of random variables X1, X2, X3, ... with the Markov property, namely that the probability of moving to the next state depends only on the present state and not on the previous states: Pr ( X n + 1 = x X 1 = x 1 , X 2 = x 2 , , X n = x n ) = Pr ( X n + 1 = x X n = x n ) , {\displaystyle \Pr(X_{n+1}=x\mid X_{1}=x_{1},X_{2}=x_{2},\ldots ,X_{n}=x_{n})=\Pr(X_{n+1}=x\mid X_{n}=x_{n}),} if both conditional probabilities are well defined, that is, if Pr ( X 1 = x 1 , , X n = x n ) > 0. {\displaystyle \Pr(X_{1}=x_{1},\ldots ,X_{n}=x_{n})>0.} The possible values of Xi form a countable set S called the state space of the chain.Time-homogeneous Markov chains are processes where Pr ( X n + 1 = x X n = y ) = Pr ( X n = x X n 1 = y ) {\displaystyle \Pr(X_{n+1}=x\mid X_{n}=y)=\Pr(X_{n}=x\mid X_{n-1}=y)} for all n. The probability of the transition is independent of n.Stationary Markov chains are processes where Pr ( X 0 = x 0 , X 1 = x 1 , , X k = x k ) = Pr ( X n = x 0 , X n + 1 = x 1 , , X n + k = x k ) {\displaystyle \Pr(X_{0}=x_{0},X_{1}=x_{1},\ldots ,X_{k}=x_{k})=\Pr(X_{n}=x_{0},X_{n+1}=x_{1},\ldots ,X_{n+k}=x_{k})} for all n and k. Every stationary chain can be proved to be time-homogeneous by Bayes' rule.A necessary and sufficient condition for a time-homogeneous Markov chain to be stationary is that the distribution of X 0 {\displaystyle X_{0}} is a stationary distribution of the Markov chain.A Markov chain with memory (or a Markov chain of order m) where m is finite, is a process satisfying Pr ( X n = x n X n 1 = x n 1 , X n 2 = x n 2 , , X 1 = x 1 ) = Pr ( X n = x n X n 1 = x n 1 , X n 2 = x n 2 , , X n m = x n m ) for n > m {\displaystyle {\begin{aligned}{}&\Pr(X_{n}=x_{n}\mid X_{n-1}=x_{n-1},X_{n-2}=x_{n-2},\dots ,X_{1}=x_{1})\\=&\Pr(X_{n}=x_{n}\mid X_{n-1}=x_{n-1},X_{n-2}=x_{n-2},\dots ,X_{n-m}=x_{n-m}){\text{ for }}n>m\end{aligned}}} In other words, the future state depends on the past m states. It is possible to construct a chain ( Y n ) {\displaystyle (Y_{n})} from ( X n ) {\displaystyle (X_{n})} which has the 'classical' Markov property by taking as state space the ordered m-tuples of X values, i.e., Y n = ( X n , X n 1 , , X n m + 1 ) {\displaystyle Y_{n}=\left(X_{n},X_{n-1},\ldots ,X_{n-m+1}\right)} .Main article: Continuous-time Markov chainA continuous-time Markov chain (Xt)t0 is defined by a finite or countable state space S, a transition rate matrix Q with dimensions equal to that of the state space and initial probability distribution defined on the state space. For ij, the elements qij are non-negative and describe the rate of the process transitions from state i to state j. The elements qii are chosen such that each row of the transition rate matrix sums to zero, while the row-sums of a probability transition matrix in a (discrete) Markov chain are all equal to one.There are three equivalent definitions of the process.[40]The continuous time Markov chain is characterized by the transition rates, the derivatives with respect to time of the transition probabilities between states i and j.Let X t {\displaystyle X_{t}} be the random variable describing the state of the process at time t, and assume the process is in a state i at time t. Then, knowing X t = i {\displaystyle X_{t}=i} , X t + h = j {\displaystyle X_{t+h}=j} is independent of previous values ( X s : s < t ) {\displaystyle \left(X_{s}:s 2 | | 3 | | n | . {\displaystyle 1=|\lambda _{1}|>|\lambda _{2}|\geq |\lambda _{3}|\geq \cdots \geq |\lambda _{n}|.} Since P is a row stochastic matrix, its largest left eigenvalue is 1. If there is a unique stationary distribution, then the largest eigenvalue and the corresponding eigenvector is unique too (because there is no other which solves the stationary distribution equation above). Let ui be the i-th column of U matrix, that is, ui is the left eigenvector of P corresponding to i. Also let x be a length n row vector that represents a valid probability distribution; since the eigenvectors ui span R n , {\displaystyle \mathbb {R} ^{n},} we can write x T = i = 1 n a i u i , a i R . {\displaystyle \mathbf {x} ^{\mathsf {T}}=\sum _{i=1}^{n}a_{i}\mathbf {u} _{i},\qquad a_{i}\in \mathbb {R} .} If we multiply x with P from right and continue this operation with the results, in the end we get the stationary distribution . In other words, = a1 u1 xPP...P = xPk as k . That means ( k ) = x ( U U 1 ) ( U U 1 ) ( U U 1 ) = x U k U 1 = ( a 1 u 1 T + a 2 u 2 T + + a n u n T ) U k U 1 = a 1 1 k u 1 T + a 2 2 k u 2 T + + a n n k u n T u i u j for i j = 1 k { a 1 u 1 T + a 2 ( 2 1 ) k u 2 T + a 3 ( 3 1 ) k u 3 T + + a n ( n 1 ) k u n T } {\displaystyle {\begin{aligned}{\boldsymbol {\pi }}^{(k)}&=\mathbf {x} \left(\mathbf {U\Sigma U} ^{-1}\right)\left(\mathbf {U\Sigma U} ^{-1}\right)\cdots \left(\mathbf {U\Sigma U} ^{-1}\right)\\&=\mathbf {xU\Sigma } ^{k}\mathbf {U} ^{-1}\\&=\left(a_{1}\mathbf {u} _{1}^{\mathsf {T}}+a_{2}\mathbf {u} _{2}^{\mathsf {T}}+\cdots +a_{n}\mathbf {u} _{n}^{\mathsf {T}}\right)\mathbf {U\Sigma } ^{k}\mathbf {U} ^{-1}\\&=a_{1}\lambda _{1}^{k}\mathbf {u} _{1}^{\mathsf {T}}+a_{2}\lambda _{2}^{k}\mathbf {u} _{2}^{\mathsf {T}}+\cdots +a_{n}\lambda _{n}^{k}\mathbf {u} _{n}^{\mathsf {T}}&&u_{i}\bot u_{j}{\text{ for }}ieq j\\&=\lambda _{1}^{k}\left\{a_{1}\mathbf {u} _{1}^{\mathsf {T}}+a_{2}\left({\frac {\lambda _{2}}{\lambda _{1}}}\right)^{k}\mathbf {u} _{2}^{\mathsf {T}}+a_{3}\left({\frac {\lambda _{3}}{\lambda _{1}}}\right)^{k}\mathbf {u} _{3}^{\mathsf {T}}+\cdots +a_{n}\left({\frac {\lambda _{n}}{\lambda _{1}}}\right)^{k}\mathbf {u} _{n}^{\mathsf {T}}\right\}\end{aligned}}} Since is parallel to u1(normalized by L2 norm) and (k) is a probability vector, (k) approaches to a1 u1 = as k with a speed in the order of 2/1 exponentially. This follows because | 2 | | n | , {\displaystyle |\lambda _{2}|\geq \cdots \geq |\lambda _{n}|,} hence 2/1 is the dominant term. The smaller the ratio is, the faster the convergence is.[44] Random noise in the state distribution can also speed up this convergence to the stationary distribution.[45]Main article: Markov chains on a measurable state spaceMany results for Markov chains with finite state space can be generalized to chains with uncountable state space through Harris chains.The use of Markov chains in Markov chain Monte Carlo methods covers cases where the process follows a continuous state space."Locally interacting Markov chains" are Markov chains with an evolution that takes into account the state of other Markov chains. This corresponds to the situation when the state space has a (Cartesian-) product form.See interacting particle system and stochastic cellular automata (probabilistic cellular automata).See for instance Interaction of Markov Processes[46]or.[47]Two states are said to communicate with each other if both are reachable from one another by a sequence of transitions that have positive probability. This is an equivalence relation which yields a set of communicating classes. A class is closed if the probability of leaving the class is zero. A Markov chain is irreducible if there is one communicating class, the state space.A state i has period k if k is the greatest common divisor of the number of transitions by which i can be reached, starting from i. That is: k = gcd { n > 0 : Pr ( X n = i X 0 = i ) > 0 } {\displaystyle k=\gcd\{n>0:\Pr(X_{n}=i\mid X_{0}=i)>0\}} The state is periodic if k > 1 {\displaystyle k>1} ; otherwise k = 1 {\displaystyle k=1} and the state is aperiodic.A state i is said to be transient if, starting from i, there is a non-zero probability that the chain will never return to i. It is called recurrent (or persistent) otherwise.[48] For a recurrent state i, the mean hitting time is defined as: M i = E [ T i ] = n = 1 n f i i ( n ) . {\displaystyle M_{i}=E[T_{i}]=\sum _{n=1}^{\infty }n\cdot f_{ii}^{(n)}.} State i is positive recurrent if M i {\displaystyle M_{i}} is finite and null recurrent otherwise. Periodicity, transience, recurrence and positive and null recurrence are class properties that is, if one state has the property then all states in its communicating class have the property.[49]A state i is called absorbing if there are no outgoing transitions from the state.Since periodicity is a class property, if a Markov chain is irreducible, then all its states have the same period. In particular, if one state is aperiodic, then the whole Markov chain is aperiodic.[50]If a finite Markov chain is irreducible, then all states are positive recurrent, and it has a unique stationary distribution given by i = 1 / E [ T i ] {\displaystyle \pi _{i}=1/E[T_{i}]} .A state i is said to be ergodic if it is aperiodic and positive recurrent. In other words, a state i is ergodic if it is recurrent, has a period of 1, and has finite mean recurrence time.If all states in an irreducible Markov chain are ergodic, then the chain is said to be ergodic. Equivalently, there exists some integer k {\displaystyle k} such that all entries of M k {\displaystyle M^{k}} are positive.It can be shown that a finite state irreducible Markov chain is ergodic if it has an aperiodic state. More generally, a Markov chain is ergodic if there is a number N such that any state can be reached from any other state in any number of steps less or equal to a number N. In case of a fully connected transition matrix, where all transitions have a non-zero probability, this condition is fulfilled withN=1.A Markov chain with more than one state and just one out-going transition per state is either not irreducible or not aperiodic, hence cannot be ergodic.Some authors call any irreducible, positive recurrent Markov chains ergodic, even periodic ones.[51] In fact, merely irreducible Markov chains correspond to ergodic processes, defined according to ergodic theory.[52]Some authors call a matrix primitive if there exists some integer k {\displaystyle k} such that all entries of M k {\displaystyle M^{k}} are positive.[53] Some authors call it regular.[54]The index of primitivity, or exponent, of a regular matrix, is the smallest k {\displaystyle k} such that all entries of M k {\displaystyle M^{k}} are positive. The exponent is purely a graph-theoretic property, since it depends only on whether each entry of M {\displaystyle M} is zero or positive, and therefore can be found on a directed graph with s i g n ( M ) {\displaystyle \mathrm {sign} (M)} as its adjacency matrix. There are several combinatorial results about the exponent when there are finitely many states. Let n {\displaystyle n} be the number of states, then[55]The exponent is ( n 1 ) 2 + 1 {\displaystyle \leq (n-1)^{2}+1} . The only case where it is an equality is when the graph of M {\displaystyle M} goes like 1 2 n 1 and 2 {\displaystyle 1\to 2\to \dots \to n\to 1{\text{ and }}2} .If M {\displaystyle M} has k 1 {\displaystyle k\geq 1} diagonal entries, then its exponent is 2 n k 1 {\displaystyle \leq 2n-k-1} .If s i g n ( M ) {\displaystyle \mathrm {sign} (M)} is symmetric, then M 2 {\displaystyle M^{2}} has positive diagonal entries, which by previous proposition means its exponent is 2 n 2 {\displaystyle \leq 2n-2} .(Dulmage-Mendelsohn theorem) The exponent is n + s ( n 2 ) {\displaystyle \leq n+s(n-2)} where s {\displaystyle s} is the girth of the graph. It can be improved to ( d + 1 ) + s ( d + 1 2 ) {\displaystyle \leq (d+1)+s(d+1-2)} , where d {\displaystyle d} is the diameter of the graph.[56]If a Markov chain has a stationary distribution, then it can be converted to a measure-preserving dynamical system: Let the probability space be = N {\displaystyle \Omega =\Sigma ^{\mathbb {N} }} , where {\displaystyle \Sigma } is the set of all states for the Markov chain. Let the sigma-algebra on the probability space be generated by the cylinder sets. Let the probability measure be generated by the stationary distribution, and the Markov chain transition. Let T : {\displaystyle T:\Omega \to \Omega } be the shift operator: T ( X 0 , X 1 , ) = ( X 1 , ) {\displaystyle T(X_{0},X_{1},\dots )=(X_{1},\dots )} . Similarly we can construct such a dynamical system with = Z {\displaystyle \Omega =\Sigma ^{\mathbb {Z} }} instead.[57]Since irreducible Markov chains with finite state spaces have a unique stationary distribution, the above construction is unambiguous for irreducible Markov chains.In ergodic theory, a measure-preserving dynamical system is called ergodic if any measurable subset S {\displaystyle S} such that T 1 ( S ) = S {\displaystyle T^{-1}(S)=S} implies S = {\displaystyle S=\emptyset } or {\displaystyle \Omega } (up to a null set).The terminology is inconsistent. Given a Markov chain with a stationary distribution that is strictly positive on all states, the Markov chain is irreducible if its corresponding measure-preserving dynamical system is ergodic.[52]In some cases, apparently non-Markovian processes may still have Markovian representations, constructed by expanding the concept of the "current" and "future" states. For example, let X be a non-Markovian process. Then define a process Y, such that each state of Y represents a time-interval of states of X. Mathematically, this takes the form: Y ( t ) = { X ( s ) : s [ a ( t ) , b ( t ) ] } . {\displaystyle Y(t)={\big \{}X(s):s\in [a(t),b(t)]\,{\big \}}.} If Y has the Markov property, then it is a Markovian representation of X.An example of a non-Markovian process with a Markovian representation is an autoregressive time series of order greater than one.[58]Main article: Phase-type distributionThe hitting time is the time, starting in a given set of states, until the chain arrives in a given state or set of states. The distribution of such a time period has a phase type distribution. The simplest such distribution is that of a single exponentially distributed transition.For a subset of states AS, the vector kA of hitting times (where element k i A {\displaystyle k_{i}^{A}} represents the expected value, starting in state i that the chain enters one of the states in the set A) is the minimal non-negative solution to[59] k i A = 0 for i A j S q i j k j A = 1 for i A . {\displaystyle {\begin{aligned}k_{i}^{A}=0&{\text{ for }}i\in A\\-\sum _{j\in S}q_{ij}k_{j}^{A}=1&{\text{ for }}iotin A.\end{aligned}}} For a CTMC Xt, the time-reversed process is defined to be X ^ t = X T t {\displaystyle {\hat {X}}_{t}=X_{T-t}} . By Kelly's lemma this process has the same stationary distribution as the forward process.A chain is said to be reversible if the reversed process is the same as the forward process. Kolmogorov's criterion states that the necessary and sufficient condition for a process to be reversible is that the product of transition rates around a closed loop must be the same in both directions.One method of finding the stationary probability distribution, , of an ergodic continuous-time Markov chain, Q, is by first finding its embedded Markov chain (EMC). Strictly speaking, the EMC is a regular discrete-time Markov chain, sometimes referred to as a jump process. Each element of the one-step transition probability matrix of the EMC, S, is denoted by sij, and represents the conditional probability of transitioning from state i into state j. These conditional probabilities may be found by s i j = { q i j k i q i k if i j 0 otherwise . {\displaystyle s_{ij}={\begin{cases}{\frac {q_{ij}}{\sum _{keq i}q_{ik}}}&{\text{if }}ieq j\\0&{\text{otherwise}}.\end{cases}}} From this, S may be written as S = I ( diag ( Q ) ) 1 Q {\displaystyle S=I-\left(\operatorname {diag} (Q)\right)^{-1}Q} where I is the identity matrix and diag(Q) is the diagonal matrix formed by selecting the main diagonal from the matrix Q and setting all other elements to zero.To find the stationary probability distribution vector, we must next find {\displaystyle \varphi } such that S = , {\displaystyle \varphi S=\varphi ,} with {\displaystyle \varphi } being a row vector, such that all elements in {\displaystyle \varphi } are greater than 0 and 1 {\displaystyle \|\varphi \|_{1}} = 1. From this, may be found as = ( diag ( Q ) ) 1 ( diag ( Q ) ) 1 1 . {\displaystyle \pi ={-\varphi (\operatorname {diag} (Q))^{-1} \over \left\|\varphi (\operatorname {diag} (Q))^{-1}\right\|_{1}}.} (S may be periodic, even if Q is not. Once is found, it must be normalized to a unit vector.)Another discrete-time process that may be derived from a continuous-time Markov chain is a -skeletonthe (discrete-time) Markov chain formed by observing X(t) at intervals of units of time. The random variables X(0),X(),X(2),... give the sequence of states visited by the -skeleton.Main article: Markov modelMarkov models are used to model changing systems. There are 4 main types of models, that generalize Markov chains depending on whether every sequential state is observable or not, and whether the system is to be adjusted on the basis of observations made:System state is fully observableSystem state is partially observableSystem is autonomousMarkov chainHidden Markov modelSystem is controlledMarkov decision processPartially observable Markov decision processMain article: Bernoulli schemeA Bernoulli scheme is a special case of a Markov chain where the transition probability matrix has identical rows, which means that the next state is independent of even the current state (in addition to being independent of the past states). A Bernoulli scheme with only two possible states is known as a Bernoulli process.Note, however, by the Ornstein isomorphism theorem, that every aperiodic and irreducible Markov chain is isomorphic to a Bernoulli scheme;[60] thus, one might equally claim that Markov chains are a "special case" of Bernoulli schemes. The isomorphism generally requires a complicated recoding. The isomorphism theorem is even a bit stronger: it states that any stationary stochastic process is isomorphic to a Bernoulli scheme; the Markov chain is just one such example.Main article: Subshift of finite typeWhen the Markov matrix is replaced by the adjacency matrix of a finite graph, the resulting shift is termed a topological Markov chain or a subshift of finite type.[60] A Markov matrix that is compatible with the adjacency matrix can then provide a measure on the subshift. Many chaotic dynamical systems are isomorphic to topological Markov chains; examples include diffeomorphisms of closed manifolds, the ProuhetThueMorse system, the Chacon system, sofic systems, context-free systems and block-coding systems.[60]Markov chains have been employed in a wide range of topics across the natural and social sciences, and in technological applications. They have been used for forecasting in several areas: for example, price trends,[61] wind power,[62] stochastic terrorism,[63][64] and solar irradiance.[65] The Markov chain forecasting models utilize a variety of settings, from discretizing the time series,[62] to hidden Markov models combined with wavelets,[61] and the Markov chain mixture distribution model (MCM).[65]Markovian systems appear extensively in thermodynamics and statistical mechanics, whenever probabilities are used to represent unknown or unmodelled details of the system, if it can be assumed that the dynamics are time-invariant, and that no relevant history need be considered which is not already included in the state description.[66][67]For example, a thermodynamic state operates under a probability distribution that is difficult or expensive to acquire. Therefore, Markov Chain Monte Carlo method can be used to draw samples randomly from a black-box to approximate the probability distribution of attributes over a range of objects.[67]Markov chains are used in lattice QCD simulations.[68] E + S E Substrate binding S E Catalytic step + P {\displaystyle {\ce {{E}+{\underset {Substrate \atop binding}{SE}}{\overset {Catalytic \atop step}{S->E}}+P}}} Michaelis-Menten kinetics. The enzyme (E) binds a substrate (S) and produces a product (P). Each reaction is a state transition in a Markov chain.A reaction network is a chemical system involving multiple reactions and chemical species. The simplest stochastic models of such networks treat the system as a continuous time Markov chain with the state being the number of molecules of each species and with reactions modeled as possible transitions of the chain.[69] Markov chains and continuous-time Markov processes are useful in chemistry when physical systems closely approximate the Markov property. For example, imagine a large number n of molecules in solution in state A, each of which can undergo a chemical reaction to state B with a certain average rate. Perhaps the molecule is an enzyme, and the states refer to how it is folded. The state of any single enzyme follows a Markov chain, and since the molecules are essentially independent of each other, the number of molecules in state A or B at a time is n times the probability a given molecule is in that state.The classical model of enzyme activity, MichaelisMenten kinetics, can be viewed as a Markov chain, where at each time step the reaction proceeds in some direction. While Michaelis-Menten is fairly straightforward, far more complicated reaction networks can also be modeled with Markov chains.[70]An algorithm based on a Markov chain was also used to focus the fragment-based growth of chemicals in silico towards a desired class of compounds such as drugs or natural products.[71] As a molecule is grown, a fragment is selected from the nascent molecule as the "current" state. It is not aware of its past (that is, it is not aware of what is already bonded to it). It then transitions to the next state when a fragment is attached to it. The transition probabilities are trained on databases of authentic classes of compounds.[72]Also, the growth (and composition) of copolymers may be modeled using Markov chains. Based on the reactivity ratios of the monomers that make up the growing polymer chain, the chain's composition may be calculated (for example, whether monomers tend to add in alternating fashion or in long runs of the same monomer). Due to steric effects, second-order Markov effects may also play a role in the growth of some polymer chains.Similarly, it has been suggested that the crystallization and growth of some epitaxial superlattice oxide materials can be accurately described by Markov chains.[73]Markov chains are used in various areas of biology. Notable examples include:Phylogenetics and bioinformatics, where most models of DNA evolution use continuous-time Markov chains to describe the nucleotide present at a given site in the genome.Population dynamics, where Markov chains are in particular a central tool in the theoretical study of matrix population models.Neurobiology, where Markov chains have been used, e.g., to simulate the mammalian neocortex.[74]Systems biology, for instance with the modeling of viral infection of single cells.[75]Compartmental models for disease outbreak and epidemic modeling.Several theorists have proposed the idea of the Markov chain statistical test (MCST), a method of conjoining Markov chains to form a "Markov blanket", arranging these chains in several recursive layers ("wafering") and producing more efficient test setssamplesas a replacement for exhaustive testing.[citation needed]Solar irradiance variability assessments are useful for solar power applications. Solar irradiance variability at any location over time is mainly a consequence of the deterministic variability of the sun's path across the sky dome and the variability in cloudiness. The variability of accessible solar irradiance on Earth's surface has been modeled using Markov chains,[76][77][78][79] also including modeling the two states of clear and cloudiness as a two-state Markov chain.[80][81]Hidden Markov models have been used in automatic speech recognition systems.[82]Markov chains are used throughout information processing. Claude Shannon's famous 1948 paper A Mathematical Theory of Communication, which in a single step created the field of information theory, opens by introducing the concept of entropy by modeling texts in a natural language (such as English) as generated by an ergodic Markov process, where each letter may depend statistically on previous letters.[83] Such idealized models can capture many of the statistical regularities of systems. Even without describing the full structure of the system perfectly, such signal models can make possible very effective data compression through entropy encoding techniques such as arithmetic coding. They also allow effective state estimation and pattern recognition. Markov chains also play an important role in reinforcement learning.Markov chains are also the basis for hidden Markov models, which are an important tool in such diverse fields as telephone networks (which use the Viterbi algorithm for error correction), speech recognition and bioinformatics (such as in rearrangements detection[84]).The LZMA lossless data compression algorithm combines Markov chains with Lempel-Ziv compression to achieve very high compression ratios.Main article: Queueing theoryMarkov chains are the basis for the analytical treatment of queues (queueing theory). Agner Krarup Erlang initiated the subject in 1917.[85] This makes them critical for optimizing the performance of telecommunications networks, where messages must often compete for limited resources (such as bandwidth).[86]Numerous queueing models use continuous-time Markov chains. For example, an M/M/1 queue is a CTMC on the non-negative integers where upward transitions from i to i+1 occur at rate according to a Poisson process and describe job arrivals, while transitions from i to i1 (for i>1) occur at rate (job service times are exponentially distributed) and describe completed services (departures) from the queue.A state diagram that represents the PageRank algorithm with a transitional probability of M, or k i + 1 N {\displaystyle {\frac {\alpha }{k_{i}}}+{\frac {1-\alpha }{N}}} .The PageRank of a webpage as used by Google is defined by a Markov chain.[87][88][89] It is the probability to be at page i {\displaystyle i} in the stationary distribution on the following Markov chain on all (known) webpages. If N {\displaystyle N} is the number of known webpages, and a page i {\displaystyle i} has k i {\displaystyle k_{i}} links to it then it has transition probability k i + 1 N {\displaystyle {\frac {\alpha }{k_{i}}}+{\frac {1-\alpha }{N}}} for all pages that are linked to and 1 N {\displaystyle {\frac {1-\alpha }{N}}} for all pages that are not linked to. The parameter {\displaystyle \alpha } is taken to be about 0.15.[90]Markov models have also been used to analyze web navigation behavior of users. A user's web link transition on a particular website can be modeled using first- or second-order Markov models and can be used to make predictions regarding future navigation and to personalize the web page for an individual user.[citation needed]Markov chain methods have also become very important for generating sequences of random numbers to accurately reflect very complicated desired probability distributions, via a process called Markov chain Monte Carlo (MCMC). In recent years this has revolutionized the practicability of Bayesian inference methods, allowing a wide range of posterior distributions to be simulated and their parameters found numerically.[citation needed]In 1971 a Naval Postgraduate School Master's thesis proposed to model a variety of combat between adversaries as a Markov chain "with states reflecting the control, maneuver, target acquisition, and target destruction actions of a weapons system" and discussed the parallels between the resulting Markov chain and Lanchester's laws.[91]In 1975 Duncan and Siverson remarked that Markov chains could be used to model conflict between state actors, and thought that their analysis would help understand "the behavior of social and political organizations in situations of conflict."[92]Markov chains are used in finance and economics to model a variety of different phenomena, including the distribution of income, the size distribution of firms, asset prices and market crashes. D. G. Champernowne built a Markov chain model of the distribution of income in 1953.[93] Herbert A. Simon and co-author Charles Bonini used a Markov chain model to derive a stationary Yule distribution of firm sizes.[94] Louis Bachelier was the first to observe that stock prices followed a random walk.[95] The random walk was later seen as evidence in favor of the efficient-market hypothesis and random walk models were popular in the literature of the 1960s.[96] Regime-switching models of business cycles were popularized by James D. Hamilton (1989), who used a Markov chain to model switches between periods of high and low GDP growth (or, alternatively, economic expansions and recessions).[97] A more recent example is the Markov switching multifractal model of Laurent E. Calvet and Adlai J. Fisher, which builds upon the convenience of earlier regime-switching models.[98][99] It uses an arbitrarily large Markov chain to drive the level of volatility of asset returns.Dynamic macroeconomics makes heavy use of Markov chains. An example is using Markov chains to exogenously model prices of equity (stock) in a general equilibrium setting.[100]Credit rating agencies produce annual tables of the transition probabilities for bonds of different credit ratings.[101]Markov chains are generally used in describing path-dependent arguments, where current structural configurations condition future outcomes. An example is the reformulation of the idea, originally due to Karl Marx's Das Kapital, tying economic development to the rise of capitalism. In current research, it is common to use a Markov chain to model how once a country reaches a specific level of economic development, the configuration of structural factors, such as size of the middle class, the ratio of urban to rural residence, the rate of political mobilization, etc., will generate a higher probability of transitioning from authoritarian to democratic regime.[102]Markov chains are employed in algorithmic music composition, particularly in software such as Csound, Max, and SuperCollider. In a first-order chain, the states of the system become note or pitch values, and a probability vector for each note is constructed, completing a transition probability matrix (see below). An algorithm is constructed to produce output note values based on the transition matrix weightings, which could be MIDI note values, frequency (Hz), or any other desirable metric.[103]1st-order matrixNoteACEA0.10.60.3C0.250.050.7E0.70.302nd-order matrixNotesADGAA0.180.60.22AD0.50.50AG0.150.750.1DD001DA0.2500.75DG0.90.10GG0.40.40.2GA0.50.250.25GD100A second-order Markov chain can be introduced by considering the current state and also the previous state, as indicated in the second table. Higher, nth-order chains tend to "group" particular notes together, while 'breaking off' into other patterns and sequences occasionally. These higher-order chains tend to generate results with a sense of phrasal structure, rather than the 'aimless wandering' produced by a first-order system.[104]Markov chains can be used structurally, as in Xenakis's Analogique A and B.[105] Markov chains are also used in systems which use a Markov model to react interactively to music input.[106]Usually musical systems need to enforce specific control constraints on the finite-length sequences they generate, but control constraints are not compatible with Markov models, since they induce long-range dependencies that violate the Markov hypothesis of limited memory. In order to overcome this limitation, a new approach has been proposed.[107]Markov chains can be used to model many games of chance. The children's games Snakes and Ladders and "Hi Ho! Cherry-O", for example, are represented exactly by Markov chains. At each turn, the player starts in a given state (on a given square) and from there has fixed odds of moving to certain other states (squares).[citation needed]Markov chain models have been used in advanced baseball analysis since 1960, although their use is still rare. Each half-inning of a baseball game fits the Markov chain state when the number of runners and outs are considered. During any at-bat, there are 24 possible combinations of number of outs and position of the runners. Mark Pankin shows that Markov chain models can be used to evaluate runs created for both individual players as well as a team.[108]He also discusses various kinds of strategies and play conditions: how Markov chain models have been used to analyze statistics for game situations such as bunting and base stealing and differences when playing on grass vs. AstroTurf.[109]Markov processes can also be used to generate superficially real-looking text given a sample document. Markov processes are used in a variety of recreational "parody generator" software (see dissociated press, Jeff Harrison,[110] Mark V. Shaney,[111][112] and Academias Neutronium). Several open-source text generation libraries using Markov chains exist.Dynamics of Markovian particlesGaussMarkov processMarkov chain approximation methodMarkov chain geostatisticsMarkov chain mixing timeMarkov chain tree theoremMarkov decision processMarkov information sourceMarkov odometerMarkov operatorMarkov random fieldMaster equationQuantum Markov chainSemi-Markov processStochastic cellular automatonTelescoping Markov chainVariable-order Markov model^ a b Sean Meyn; Richard L. Tweedie (2 April 2009). Markov Chains and Stochastic Stability. Cambridge University Press. p.3. ISBN978-0-521-73182-9.^ Reuven Y. Rubinstein; Dirk P. Kroese (20 September 2011). Simulation and the Monte Carlo Method. John Wiley & Sons. p.225. ISBN978-1-118-21052-9.^ Dani Gamerman; Hedibert F. Lopes (10 May 2006). 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ISBN978-0-7923-9650-5."Markov chain", Encyclopedia of Mathematics, EMS Press, 2001 [1994]Markov Chains chapter in American Mathematical Society's introductory probability book Archived 2008-05-22 at the Wayback MachineIntroduction to Markov Chains on YouTubeA visual explanation of Markov ChainsOriginal paper by A.A Markov (1913): An Example of Statistical Investigation of the Text Eugene Onegin Concerning the Connection of Samples in Chains (translated from Russian)Retrieved from " Markov chains, named after Andrey Markov, a stochastic model that depicts a sequence of possible events where predictions or probabilities for the next state are based solely on its previous event state, not the states before. In simple words, the probability that n+1th steps will be x depends only on the nth steps not the complete sequence of steps that came before n. This property is known as Markov Property or Memorylessness. Let us explore our Markov chain with the help of a diagram, are Markov Process A diagram representing a two-state(here, E and A) Markov process. Here the arrows originated from the current state and point to the future state and the number associated with the arrows indicates the probability of the Markov process changing from one state to another state. For instance, if the Markov process is in state E, then the probability it changes to state A is 0.7, while the probability it remains in the same state is 0.3. Similarly, for any process in state A, the probability to change to Estate is 0.4 and the probability to remain in the same state is 0.6. How to Represent Markov Chain? From the diagram of the two-state Markov process, we can understand that the Markov chain is a directed graph. So we can represent it with the help of an adjacency matrix. +------+------+ | A | E | --- Each element denotes the probability weight of the edge +------+------+------+ connecting the two corresponding vertices | A | 0.6 | 0.4 | --- 0.4 is the probability for state A to go to state E and 0.6 is the probability +------+------+------+ to remain at the same state | E | 0.7 | 0.3 | --- 0.7 is the probability for state E to go to state A and 0.3 is the probability +------+------+------+ to remain at the same state This matrix is also called Transition Matrix. If the Markov chain has N possible states, the matrix will be an NxN matrix. Each row of this matrix should sum to 1. In addition to this, a Markov chain also has an Initial State Vector of order Nx1. These two entities are a must to represent a Markov chain. N-step Transition Matrix: Now let us learn higher-order transition matrices. It helps us to find the chance of that transition occurring over multiple steps. To put in simple words, what will be the probability of moving from state A to state E over the N step? There is actually a very simple way to calculate it. This can be determined by calculating the value of entry (A, ) of the matrix obtained by raising the transition matrix to the power of N. Types of Markov Chain : discrete-time Markov chains : This implies the index set T( state of the process at time t ) is a countable set here or we can say that changes occur at specific states. Generally, the term "Markov chain" is used for DTMC. continuous-time Markov chains: Here the index set T( state of the process at time t ) is a continuum, which means changes are continuous in CTMC. Properties of Markov Chain :A Markov chain is said to be Irreducible if we can go from one state to another in a single or more than one step.A state in a Markov chain is said to be Periodic if returning to it requires a multiple of some integer larger than 1, the greatest common divisor of all the possible return path lengths will be the period of that state.A state in a Markov chain is said to be Transient if there is a non-zero probability that the chain will never return to the same state, otherwise, it is Recurrent.A state in a Markov chain is called Absorbing if there is no possible way to leave that state. Absorbing states do not have any outgoing transitions from it.Markov Chain in Python : Python3 # let's import our libraryimport scipy.linalgimport numpy as np # Encoding this states to numbers as it# is easier to deal with numbers instead # of words.state = ["A", "E"] # Assigning the transition matrix to a variable# i.e a numpy 2d matrix.MyMatrix = np.array([[0.6, 0.4], [0.7, 0.3]]) # Simulating a random walk on our Markov chain # with 20 steps. Random walk simply means that# we start with an arbitrary state and then we# move along our markov chain.n = 20 # decide which state to start withStartingState = 0CurrentState = StartingState # printing the stating state using state# dictionaryprint(state[CurrentState], "--->", end=" ") while n-1: # Deciding the next state using a random.choice() # function,that takes list of states and the probability # to go to the next states from our current state CurrentState = np.random.choice([0, 1], p=MyMatrix[CurrentState]) # printing the path of random walk print(state[CurrentState], "--->", end=" ") n -= 1print("stop") # Let us find the stationary distribution of our # Markov chain by Finding Left Eigen Vectors# We only need the left eigen vectorsMyValues, left = scipy.linalg.eig(MyMatrix, right=False, left=True) print("left eigen vectors = ", left, "")print("eigen values = ", MyValues) # Pi is a probability distribution so the sum of # the probabilities should be 1 To get that from # the above negative values we just have to normalizepi = left[:, 0]pi_normalized = [(x/np.sum(pi)).real for x in pi]pi_normalizedApplication of Markov Chain : Markov chains make the study of many real-world processes much more simple and easy to understand. Using the Markov chain we can derive some useful results such as Stationary Distribution and many more. MCMC(Markov Chain Monte Carlo), which gives a solution to the problems that come from the normalization factor, is based on Markov Chain.Markov Chains are used in information theory, search engines, speech recognition etc.Markov chain has huge possibilities, future and importance in the field of Data Science and the interested readers are requested to learn this stuff properly for being a competent person in the field of Data Science.Assumptions for Markov Chain :The statistical system contains a finite number of states.The states are mutually exclusive and collectively exhaustive.The transition probability from one state to another state is constant over time. Markov processes are fairly common in real-life problems and Markov chains can be easily implemented because of their memorylessness property. Using Markov chain can simplify the problem without affecting its accuracy. Let us take an example to understand the advantage of this tool, suppose my friend is suggesting to have a meal. I may say that I do not want a pizza as I have that one hour ago. But Is it appropriate if I say that I do not want a pizza because I have it two months ago? That means in this case, my probability of picking a meal is entirely dependent on my immediately preceding meal. Here is the effectiveness of the Markov Chain.

**Markov chain simple example. What is markov chain model. Markov chains & simulation techniques. Markov chain. What is markov chain analysis. What is markov chain monte carlo simulation. Markov chain simulation. What is markov chain used for. How does markov chain work.**