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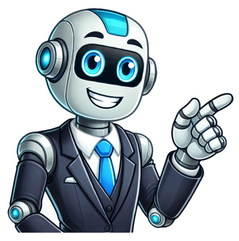

Home>Arithmetic sequence – Pattern, Formula, and ExplanationSolutionTo confirm whether a sequence is an arithmetic sequence, we can check the sequences' patterns and see if each pair of consecutive terms shares a common difference. If they do, the sequence is considered an arithmetic sequence.\begin{aligned} 3^{\color{green}\displaystyle{\overset{+6}{\rightarrow}}} 9^{\color{green}\displaystyle{\overset{+6}{\rightarrow}}} 15^{\color{green}\displaystyle{\overset{+6}{\rightarrow}}} … 75^{\color{green}\displaystyle{\overset{+6}{\rightarrow}}} 81\end{aligned}We can see that the first sequence share a common difference of 6, so the given sequence is an arithmetic sequence.Applying the same process for the next sequence, we have the following:\begin{aligned} -12^{\color{green}\displaystyle{\overset{+6}{\rightarrow}}} -6^{\color{green}\displaystyle{\overset{+6}{\rightarrow}}} 0^{\color{green}\displaystyle{\overset{+6}{\rightarrow}}} … 60^{\color{green}\displaystyle{\overset{+6}{\rightarrow}}} 66\end{aligned}Hence, the second sequence is also an arithmetic sequence.\begin{aligned} 2^{\color{green}\displaystyle{\overset{+2}{\rightarrow}}} -4^{\color{green}\displaystyle{\overset{+4}{\rightarrow}}} 8^{\color{green}\displaystyle{\overset{+8}{\rightarrow}}} … 256^{\color{green}\displaystyle{\overset{+256}{\rightarrow}}} 512\end{aligned}The differences between the pairs of consecutive terms vary, so the sequence is not an arithmetic sequence.Let's now take a look at the last sequence:\begin{aligned} 2^{\color{green}\displaystyle{\overset{+3}{\rightarrow}}} -5^{\color{green}\displaystyle{\overset{+3}{\rightarrow}}} 8^{\color{green}\displaystyle{\overset{+5}{\rightarrow}}} … 71^{\color{green}\displaystyle{\overset{+4}{\rightarrow}}} 75\end{aligned}Since the pairs of consecutive terms have varying differences, so the sequence is not an arithmetic sequence.Example 2What is the common difference of the arithmetic sequence, $-9, -3, 3, 9, …$? Find the seventh term of the sequence.SolutionWe can start by observing the common difference shared by each pair of consecutive terms.$-3 – (-9) = 6$$3 – (-3) = 6$$9 – 3 = 6$This means that the arithmetic sequence has a common difference of 6. We can use the common difference to find the next terms by adding each term by 6 until we have the seventh term.\begin{aligned} a_4&= 9\\ a_5&= 9+ 6\\&= 15\\a_6 &= 15 + 6\\&= 21\\a_7 &= 21 + 6\\&= 27\end{aligned}This means that the seventh term of the arithmetic sequence is 27.Example 3Find an equation that represents the general term, a_n, of the given arithmetic sequence, $12, 6, 0, -6, -12, …$. Use this equation to find the 100th term of the sequence.SolutionWe can begin by finding the common difference of the arithmetic sequence.\begin{aligned} 12^{\color{green}\displaystyle{\overset{-6}{\rightarrow}}} 6^{\color{green}\displaystyle{\overset{-6}{\rightarrow}}} 0^{\color{green}\displaystyle{\overset{-6}{\rightarrow}}} -6^{\color{green}\displaystyle{\overset{-6}{\rightarrow}}} -12\end{aligned}From this, we can see that the sequence has a common difference, d, of -6. To find the equation representing the nth term of the sequence, we can use the explicit rule for the arithmetic sequence, as shown below.\begin{aligned} a_n &= a_1 + (n-1)d\\&= 12 + (n – 1)(-6)\\&= 12 + (-6n + 6)\\&= 12 – 6n + 6\\&= 18 – 6n\end{aligned}Now that we have the equation for the nth term's general form, we substitute $n = 100$ into the equation to find the hundredth term.\begin{aligned} a_{100}&= 18 – 6(100)\\&=18 – 600\\&= -582\end{aligned}This means that the general form of sequence's nth term is $-6n + 18$, and the 100th term of the sequence is -582.Example 4What is the sum of the series $6 + 11 + 16 + 21 + … +216+ 221$?Solution\begin{aligned} 6^{\color{green}\displaystyle{\overset{+5}{\rightarrow}}} 11^{\color{green}\displaystyle{\overset{+5}{\rightarrow}}} 16^{\color{green}\displaystyle{\overset{+5}{\rightarrow}}} 21^{\color{green}\displaystyle{\overset{+5}{\rightarrow}}} …,216^{\color{green}\displaystyle{\overset{+5}{\rightarrow}}} 221\end{aligned}Observing the sequence, we can see that the common difference, d, of the series is equal to 5.To find the sum of the arithmetic series, we'll need the first and last terms and the number of terms within the series. Let's use the explicit rule, $a_n = a_1 + (n – 1)d$, to find the value of n first.\begin{aligned} a_1 &= 6\\a_n&= 221\\d&= 5\\\221 &= 6 + (n-1)5\\215&=5(n – 1)\\43&= n – 1\\ n&= 44\end{aligned}This means that the series has 44 terms. We can now use the formula for the sum of an arithmetic series since we have a_1, a_n, and n already.\begin{aligned} S_n &= \dfrac{n(a_1 + a_n)}{2}\\&=\dfrac{44(6 + 221)}{2}\\&= \dfrac{44(227)}{2}\\&= 22(227)\\&= 4994\end{aligned}Hence, the arithmetic series has a sum of 4994.Example 5 An outdoor amphitheater contains 12 seats on its first row, 14 seats on the second row, 16 seats on the third row, etc. If the amphitheater has twelve rows, how can seats can it accommodate?SolutionAs we can observe, the outdoor amphitheater increases gradually over each row.\begin{aligned} 12^{\color{green}\displaystyle{\overset{+2}{\rightarrow}}} 14^{\color{green}\displaystyle{\overset{+2}{\rightarrow}}} 16…\end{aligned}Since each pair of consecutive terms has 2 as its common difference, we can say that the number of seats for all eighteen rows will reflect an arithmetic sequence.We'll need to know the last term of the sequence first, and we can use the explicit rule to find the number of seats in the 18th row.\begin{aligned} a_n &= a_1 + (n-1)d \\ a_{18} &= 12 + (18 – 1)2\\&= 12 + 34\\&= 46 \end{aligned}This means that the 18th row has 46 seats. To find the total number of seats, we can find the sum of the entire sequence (or the arithmetic series) using the formula, $S_n = \dfrac{n(a_1 + a_n)}{2}$.\begin{aligned} 12 + 14 + 16 + … + 46 &= S_n\\&=\dfrac{18(12 + 46)}{2}\\&= \dfrac{18(58)}{2}\\&=9 (58)\\&= 522\end{aligned}This means that the outdoor amphitheater has a total seat capacity of 522. An arithmetic sequence or progression is defined as a sequence of numbers in which the difference between one term and the next term remains constant.For example: the given below sequence has a common difference of 1.1 2 3 4 5 . . . n ⇑ ⇑ ⇑ ⇑ ⇑ ⇑ . . . 1st 2nd 3rd 4th 5th . . . nth TermsThe Arithmetic Sequence can be represented as:Arithmetic Sequence / Arithmetic ProgresionThe above image has a common difference of 2.Note: A negative common difference causes the sequence to decrease, and this is entirely valid in an arithmetic progression (AP).General Formula for the Nth TermThe general formula to find the nth term of an arithmetic sequence is:an = a1 + (n − 1) · dWhere:an = nth term,a = first term,d = common difference,n = term number.Example: Nth Term of Arithmetic SequenceLet's consider an example of Arithmetic Sequence 6, 16, 26, 36, 46, 56, 66, . . .Term (n)ExpressionValuea16 + (1 - 1) · 106a26 + (2 -1) · 10 = 6 + 1016a36 + (3 − 1) · 10 = 6 + 10 + 1026a46 + (4 - 1) · 10 = 6 + 10 + 10 + 1036a56 + (5 − 1) · 10 = 6 + 10 + 10 + 10 + 1046a66 + (6 - 1) · 10 = 6 + 10 + 10 + 10 + 10 + 1056a76 + (7 − 1) · 10 = 6 + 10 + 10 + 10 + 10 + 10 + 1066General Formula for the above sequence:Substituting the values a = 6 and d = 10 into the formula: an = 6 + (n - 1) × 10This formula can be used to find any term of the sequence directly. For example:a10 = 6 + (10 − 1) · 10 = 6 + 90 = 96,a22 = 6 + (22 − 1) · 10 = 6 + 210 = 216.Examples of Arithmetic Sequence Here are some examples of arithmetic sequences,1. Sequence of Natural Numbers1, 2, 3, 4, 5, 6, . . .First term a1 = 1Common difference (d): 2 - 1 = 1.2. Sequence of Even Numbers2, 4, 6, 8, 10, 12, . . .First term a1 = 2Common difference (d): 4 - 2 = 2.3. Sequence of Odd Numbers1, 3, 5, 7, 9, 11, . . .First term a1 = 1Common difference (d): 3 - 1 = 2.4. Negative Arithmetic Sequence:−5, −10, −15, −20, -25, -30, . . .First term a1 = -5Common difference (d): -10 - (-5) = -10 + 5 = -5.Read more: How to Find the Nth term of Arithmetic Sequence?Sum of terms in Arithmetic SequenceThe sum of terms of an Arithmetic Sequence is calledan Arithmetic Series.Suppose the first term of the arithmetic series is a and the common difference is d then the sum of the n term of this arithmetic series is given using the formula,Sn= n/2 [2a + (n - 1)d]If the common difference of the arithmetic series is not given but the last term of the series is given (say l). Then its sum is calculated as,Sn= n/2 [a + l]Example: A tree fruits five apples in the first year and at each successive year it has 2 more apples than the last year find the total apple the tree bears at the end of six years.Solution:Apple bear by tree in first year (a) = 5Yearly Increase in apple bear by the tree (d) = 2Time Period = 6 yearsTotal apple at the end of five years in the tree.Sn = n/2 [2a + (n-1)d]⇒ Sn = 6/2(2(5) + (6 - 1)(2))⇒ Sn = 6/2 (10 + 10)⇒ Sn = 3(20)⇒ Sn = 60Thus, the apple in the tree at the end of six-year is 60 applesRecursive Formula for Arithmetic SequenceThe nth term of an Arithmetic Sequence can be defined recursively as the next term can always be obtained by adding a common difference to the preceding term, the following derivation can be used to illustrate the same thing.As we know, nth term of the Arithmetic Sequence is given by an = a + (n - 1) × dthus, (n - 1)th term can be given by an-1 = a + (n - 1 - 1) × dan-1 = a + (n - 2) × dThus, an = a + (n - 1) × d⇒ an = a + (n - 1 - 1 + 1) × d⇒ an = a + (n - 2 + 1) × d ⇒ an = a + (n - 2) × d + d⇒ an = an-1 + dImportant Formulas & Notations in Arithmetic ProgressionThe notations used in arithmetic progression are :First Term → aCommon Difference → dNth term → anSum of First n Terms → SnThe following table contains formulas related to Arithmetic Progression.Common Differencean+1 - an = d General Form of APa, a + d, a + 2d, a + 3d, . . .nth term of APan = a (n – 1) × dSum of the n terms in APS = n/2[2a + (n − 1) × d]Sum of all terms in a finite AP with the last term as 'l'n/2(a + l)Solved Problems on Arithmetic SequenceQuestion 1: Find the AP if the first term is 15 and the common difference is 4.Solution:As we know,a, a + d, a + 2d, a + 3d, a + 4d, ...Here, a = 15 and d = 4= 15, (15 + 4), (15 + 2 × 4), (15 + 3 × 4), (15 + 4 × 4),= 15, 19, (15 + 8), (15 + 12), (15 + 16), ... = 15, 19, 23, 27, 31, ...and so on.So the AP is 15, 19, 23, 27, 31...Question 2: Find the 20th term for the given AP: 3, 5, 7, 9, ...Solution: Given, 3, 5, 7, 9, 11, . . .a = 3, d = 5 – 3 = 2, n = 20Thus, an = a + (n − 1)d⇒ a20 = 3 + (20 − 1)2⇒ a20 = 3 + 38⇒ a20 = 41Here 20th term is a20 = 41Question 3: Find the sum of the first 20 multiples of 5.Solution:First 20 multiples of 5 are 5, 10, 15, . . . , 100.Here, it is clear that the sequence formed is an AP where,a = 5, d = 5, an = 100, n = 20.Thus, Sn = n/2 [2a + (n − 1) d]⇒ Sn = 20/2 [2 × 5 + (20 − 1)5]⇒ Sn = 10 [10 + 95]⇒ Sn = 1050People Also Read: Share — copy and redistribute the material in any medium or format for any purpose, even commercially. Adapt — remix, transform, and build upon the material for any purpose, even commercially. The licensor cannot revoke these freedoms as long as you follow the license terms. Attribution — You must give appropriate credit , provide a link to the license, and indicate if changes were made . You may do so in any reasonable manner, but not in any way that suggests the licensor endorses you or your use. ShareAlike — If you remix, transform, or build upon the material, you must distribute your contributions under the same license as the original. No additional restrictions — You may not apply legal terms or technological measures that legally restrict others from doing anything the license permits. You do not have to comply with the license for elements of the material in the public domain or where your use is permitted by an applicable exception or limitation . No warranties are given. The license may not give you all of the permissions necessary for your intended use. For example, other rights such as publicity, privacy, or moral rights may limit how you use the material. After reading this article, you will be able to: define an arithmetic sequence;find the common difference of an arithmetic series;determine the nth term of an arithmetic series; formulate a general formula for the nth term of any arithmetic series;differentiate arithmetic sequence from arithmetic series;solve for the arithmetic series; andknow the importance of learning arithmetic series. An arithmetic progression or sequence is a collection of numbers in which the difference between consecutive terms is a constant. It is said to be an arithmetic sequence if the sequence is created by adding or subtracting the same number each time. In a nutshell, it is a set of numbers that is in order. For example, the sequence 3, 18, 13, 18, 23, 28, 33 is an arithmetic progression with a common difference of 5. The formal definition of an arithmetic sequence, or arithmetic progression, defines it as a sequence of numbers in which each subsequent number is the sum of the preceding number and some constant d. Mathematically, an arithmetic sequence is in the form a, a + d, a + 2d, a + 3d, a + 4d, a + 5d, ... where,a is the first term of the sequence; andd is the common difference of the sequence. There are specific terms used in an arithmetic sequence that you need to remember as it will help us solve problems involving arithmetic sequence. Terms – in an arithmetic sequence, each number is called as a term. Alternatively, we can also call these numbers as elements or members. Say, for example, in the sequence of 3, 8, 13, 18, 23, 28, and 33. Each number is named based on their order in the sequence. First term – in an arithmetic sequence, the first term, as the name implies, is the initial term in the sequence. It is denoted by a1 or a. Say, for example, in the sequence of 3, 8, 13, 18, 23, 28, and 33, the first term is 3. When solving problems involving arithmetic sequence, we can denote it as a1 = 3 or a = 3. Common difference – the common difference of an arithmetic sequence is the difference between two consecutive terms of an arithmetic progression. For example, in the sequence of 3, 8, 13, 18, 23, 28, and 33, the common difference is determined by subtracting: 8 – 3 = 5; 13 – 8 = 5; 18 – 13 = 5; 23 – 18 = 5; 28 – 23 = 5; 33 – 38 = 5. We can show the common difference of an arithmetic sequence as: or as: Since the difference of all the two consecutive terms of the sequence is equal, we can say that the common difference is 5. nth term – the nth term of an arithmetic sequence is any term of the sequence which is not given in the sequence. The nth term of an arithmetic sequence is defined by the first term and the common difference of the sequence. By definition, the common difference of an arithmetic sequence is the fixed number you add or subtract to find the following number of the sequence. Usually, we use the first and second terms of the arithmetic progression to find the common difference. Thus, d= a_2-a_1 where, d is the common difference of the sequence; a_1 is the first term of the sequence; and a_2 is the first term of the sequence. Generally, we can define the common difference of the sequence as: d= a_m-a_{m-1} where, d is the common difference of the sequence; a_m is the any term in the arithmetic sequence; and a_{m-1} is the term before a_m. Example #1 What is the common difference of the arithmetic sequence of 6, 9, 12, 15, 18, 21? Solution Step 1: Simply use the formula of d= a_2-a_1. Thus, d = 9 – 6 d = 3 Step 2: Check if it is true for all the succeeding terms. Thus, – 9 = 3; 15 – 12 = 3; 18 – 15 = 3; 21 – 15 = 3 Since 3 is the difference of all the consecutive terms, we can now conclude that the common difference is 3. Therefore, the common difference of the sequence 6, 9, 12, 15, 18, 21 is 3. Example #2 Determine the common difference of the arithmetic sequence __, 4, 10, 16, __, 28, 34, 40. Solution Step 1: Since there are missing terms in the given arithmetic sequence, we will use the formula d= am-am-1, wherein we can choose any two consecutive terms in the given arithmetic progression. Thus, d = 16 – 10 d = 6 Step 2: Check if it is true for all the consecutive terms. Thus, 10 – 4 = 6; 34 – 28 = 6; 40 – 34 = 6 Since the result of subtracting all two consecutive terms is 6, we can conclude that it is their common difference. Therefore, the common difference of the arithmetic sequence is 6. Usually, the nth term of an arithmetic sequence is defined by the first term and the arithmetic progression's common difference. Thus, $a_n = a_1 + (n – 1) d$ where, a_n is the nth term of the arithmetic sequence; a_1 is the first term of the arithmetic sequence; n is the position of the nth term in the arithmetic sequence; and d is the common difference of the arithmetic sequence. Generally, we can denote it as: $a_n = a_m + (n – m) d$ where, a_n is the nth term of the arithmetic sequence; a_m is any term in the arithmetic sequence; n is the position of the nth term in the arithmetic sequence; m is the position of the mth term in the arithmetic sequence; and d is the common difference of the arithmetic sequence. Example #1 What is the 7th term of the arithmetic sequence if the first term is 5 and the common difference is 13? Solution Step 1: List all the information given in the problem. $a_n = a_7; a_1 = 5$; d = 13; n = 7 Step 2: To find the 7th term of the arithmetic progression, use the formula $a_n = a_1 + (n – 1) d$. By substitution, we will have: $a_n = a_1 + (n – 1) da_7 = 5 + (7 – 1) 5a_7 = 5 + (6) 5 a_7 = 5 + 30a_7 = 35$ Therefore, the seventh term of the arithmetic progression is 35. Example #2 Determine the 12th term of the arithmetic sequence if the fifth term is 23 and the common difference is 7. Solution Step 1: List all the information given in the problem. $a_n = a_{12}; a_5 = 23$; d = 7; n = 12; m = 5 Step 2: To find the 12th term of the arithmetic progression, use the formula $a_n = a_m + (n – m) d$. By substitution, we will have: $a_n = a_m + (n – m) da_{12} = 23 + (12 – 5) 7a_{12} = 23 + (7) 7a_{12} = 23 + 49a_{12} = 72$ Example #3 Find the common difference and the unknown terms of the arithmetic sequence shown below. 9, 13, __, 21, 25, 29, __, 37, 41, ... Solution Step 1: Determine the common difference of the arithmetic sequence. Thus, d = 13 – 9 d = 4 Step 2: The missing unknown terms are the 3rd and 7th terms. Since the 2nd and 6th term of the arithmetic sequence is shown, we can simply add the common difference to the preceding term of the unknown terms. Thus; $a_3 = a_2 + d$ $a_3 = 13 + 4a_3 = 17$ $a_7 = a_6 + d$ $a_7 = 29 + 4a_7 = 33$ Therefore, the common difference of the arithmetic progression is 4. The third and seventh term is 17 and 33, respectively. Example #4 Find the common difference and the 20th term of the arithmetic sequence described below. -21, -15, -9, -3, 3, ... Solution Step 1: Determine the common difference of the arithmetic progression. Thus, d = -15 – (-21) d = -15 + 21d = 6 Step 2: List all the information necessary to solve the problem. $a_n = a_{20}; a_1 = -21$; d = 6; n = 20 Step 3: To find the 7th term of the arithmetic progression, use the formula $a_n = a_1 + (n – 1) d$. Thus, by substitution, $a_n = a_1 + (n – 1) da_{20} = -21 + (20 – 1) 6a_{20} = -21 + (19) 6 a_{20} = -21 + 114a_{20} = 93$ Therefore, the common difference is 6 and the 20th term is 93. The formula of finding the nth term, $a_n = a_1 + (n – 1) d$, yields the general form of the arithmetic progression as long as the first term and the common difference is known. Say, for example, the sequence -21, -15, -9, -3, 3, ... have a common difference that is 6. Then, using the formula, the general form of the sequence -21, -15, -9, -3, 3, ... will be: $a_n = a_1 + (n – 1) da_n = -21 + (n – 1) 6a_n = -21 + (6n – 6)a_n = 6n – 27$ Therefore, the general formula in finding the nth term of the sequence is -21, -15, -9, -3, 3, ... is $a_n = 6n – 27.$ Let's look at the other examples below. Arithmetic SequenceFirst TermCommon DifferenceGeneral Form of the nth term3, 8, 13, 18, 23, ...35$a_n = 5n – 2$6, -1, -8, -15, -22, ...6-7$a_n = 13 – 7n$$\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4},...$ $\frac{1}{4}$$\frac{1}{2}$$a_n = \frac{1}{2}n – \frac{1}{4}$$\pi, 3\pi , 5\pi , 7\pi , 9\pi$ π2$\pi$$a_n = 2\pi{n} – \pi$ $\sqrt{5} , 5\sqrt{5} , 9\sqrt{5} , 13 \sqrt{5}$ $\sqrt{5}$$4\sqrt{5}$$a_n = (4\sqrt{5})n-3\sqrt{5}$ An arithmetic series is the sum of the terms of a finite arithmetic progression. The arithmetic series or sum of the arithmetic sequence is defined by the formula $S_n= \frac{n(a_1 + a_n)}{2}$ where, S_n is the sum of the finite arithmetic progression; a_1 is the first term of the finite arithmetic progression; a_n is the last term of the finite arithmetic progression; and n is the number of terms in the finite arithmetic progression. To derive the formula $S_n= \frac{n(a_1 + a_n)}{2}$, begin by expressing the arithmetic sequence in two different ways such that: $S_n= a_1 + (a_1 + d) + (a_1 + 2d)+... +[a_1 + (n – 2) d] + [a_1 + (n – 1) d]$ $S_n=[a_n – (n – 1) d]+ [a_n – (n – 2) d] +...+[a_n- 2d+ a_n- d + a_n$ Then, by adding the two arithmetic sequence, all terms involving d will be cancelled out. Thus, having the equation: $2S_n = n (a_1 + a_n)$ Dividing both sides of the equation by 2 will result to: $S_n = \frac{n (a_1 + a_n)}{2}$ Alternatively, if substitute the value of a_n to $a_n = a_1 + (n – 1) d$, we will have the form: $S_n = \frac{n}{2}[2{a_1} + (n+1)d]$ To solve for the arithmetic series or the sum of an arithmetic sequence: Determine the first term and common difference of the arithmetic sequence.Use the formula $S_n = \frac{n (a_1 + a_n)}{2}$ in finding the sum. Example #1 What is the sum of the finite arithmetic sequence 21, 29, 37, 45, 53, 61? Solution Step 1: Determine the common difference of the arithmetic progression. Thus, d = 29 – 21 d = 8 Step 2: List all the necessary information. n = 6, a_1 = 21, a_6 = 61, d = 8 Step 3: Use the formula $S_n = \frac{n (a_1 + a_n)}{2}$ to solve for the arithmetic series. By substitution, the sum is: $S_n = \frac{n (a_1 + a_n)}{2} S_6= \frac{6(21 + 61)}{2} S_6 = \frac{6(82)}{2} S_6 = \frac{492}{2} S_6 = 246$ Therefore, the sum of the finite arithmetic progression is 246. Example #2 What is the sum of the first 50 counting numbers? Solution Step 1: Determine the common difference of the arithmetic sequence. Since the problem states that the we are looking for the sum of the first consecutive counting numbers, thus, d = 1. Step 2: List all the necessary information. n = 50, a_1 = 1, a_{50} = 50, d = 1 Step 3: Use the formula $S_n = \frac{n (a_1 + a_n)}{2}$ to solve for the arithmetic series. By substitution, the sum is: $S_n = \frac{n (a_1 + a_n)}{2} S_{50}= \frac{50(1 + 50)}{2} S_{50} = \frac{50(51)}{2} S_{50} = \frac{2550}{2} S_{50} = 1275$ Therefore, the sum of the first 50 counting numbers is 1275. Example #3 If we make an arithmetic progression that starts counting the year from 1990 until 2021, what will be the sum of this arithmetic progression? Solution Step 1: Determine the common difference of the arithmetic sequence. Since the problem states that the we are looking for the sum of the arithmetic progression if we count by years, thus, d = 1. Step 2: List all the necessary information. n = 32, $a_1 = 1990, a_{32} = 2021$, d = 1 Step 3: Use the formula $S_n = \frac{n (a_1 + a_n)}{2}$ to solve for the arithmetic series. By substitution, the sum is: $S_n = \frac{n (a_1 + a_n)}{2} S_{32}= \frac{32(1990+ 2021}{2} S_{32} = \frac{32(4011)}{2} S_{32} = \frac{128 532}{2} S_{32} = 64 176$ Therefore, the sum of the arithmetic progression is 64 176. Mathematicians were inspired by nature to attempt to explain these patterns in nature, to develop mathematical models, and gain a basic understanding of geometric shapes and structures. In real life, the arithmetic sequence is essential because it enables us to understand concepts through patterns. An arithmetic sequence is an excellent starting point for describing various things, such as time, which has a common difference of one hour. An arithmetic sequence is also necessary for simulating recurring events. Additionally, mathematical sequences and series are used in: stacking cups, chairs, bowls;pyramid-like patterns in which objects are constantly increasing or decreasing in size;business and financial analysis to aid in decision-making;determine the optimal solution to a given problem; andquantitative analysis is used by organizations to perform risk assessment and management, investment decision-making, and pricing, among other critical functions. Number Patterns and Sequence 4th Grade Math WorksheetsMeasures of Time (days, weeks, months, and years) (Education Themed) WorksheetsNumber Patterns (Farm Themed) Worksheets We spend a lot of time researching and compiling the information on this site. If you find this useful in your research, please use the tool below to properly link to or reference Helping with Math as the source. We appreciate your support! An arithmetic sequence is a sequence where each term is found by adding or subtracting the same value from one term to the next. This value that is added or subtracted is called "common sum" or "common difference" If the common difference is positive, the terms of the sequence will increase in value.If the common difference is negative, the terms of the sequence will decrease in value. For example, the following two sequences are examples of arithmetic sequences. 1, 4, 7, 10, 13, 16, 19, ...70, 62, 54, 48, 40, ... Looking carefully at 1, 4, 7, 10, 13, 16, 19, ..., helps us to make the following observation:As you can see, each term is found by adding 3, a common sum, to the previous term. Looking carefully at 70, 62, 54, 46, 38, ..., helps us to make the following observation:This time, to find each term, we subtract 8, a common difference from the previous term. Many arithmetic sequences can me modeled with an algebraic expression Here is a trick or "recipe per se" to quickly get an algebraic expression!1) Let us try to model 1, 4, 7, 10, 13, 16, 19, ...Let n represent any term number in the sequence. The number we add to each term is 3.The number that comes right before 1 in the sequence is -2.We can therefore model the sequence with this algebra expression: 3 × n + -2.Check to see if the algebra expression works:When n = 1, which represents the first term, we get 3 × 1 + -2 = 3 + -2 = 1When n = 2, which represents the second term, we get 3 × 2 + -2 = 6 + -2 = 4When n = 3, which represents the third term, we get 3 × 3 + -2 = 9 + -2 = 7 The algebraic expression works! 2) Let us try to model 70, 62, 54, 46, 38, ... Let n represent any term number in the sequence. The number we subtract to each term is -8.The number that comes right before 70 in the sequence is 78.We can therefore model the sequence with this algebraic expression: -8 × n + 78.Check to see if the algebra expression works:When n = 1, which represents the first term, we get -8 × 1 + 78 = -8 + 78 = 70When n = 2, which represents the second term, we get -8 × 2 + 78 = -16 + 78 = 62When n = 3, which represents the second term, we get -8 × 3 + 78 = -24 + 78 = 54 Again, the algebraic expression works! Arithmetic sequence formula The way that we modeled the arithmetic sequences above with algebraic expressions is a shortcut. We will now look for the arithmetic sequence formula using the algebraic expressions.1)3 × n + -2 is the algebraic expression for 1, 4, 7, 10, 13, 16, 19, ...Let us try to rewrite 3 × n + -2 by making the first term appear in the expression.3 × n + -2 = 3 × n + -3 + 1 (since -2 = -3 + 1)3 × n + -2 = 3 × (n - 1) + 13 is the number we add to each term1 is the first termn is the number of terms2)-8 × n + 78 is the algebraic expression for 70, 62, 54, 46, 38, ...Let us try to rewrite -8 × n + 78 by making the first term appear in the expression.-8 × n + 78 = -8 × n + 8 + 70 (since 78 = 8 + 70)-8 × n + 78 = -8 × (n - 1) + 70-8 is the number we add to each term70 is the first termn is the number of terms In general, Let d be the number we add each time or the common difference.Let a1 be the first termLet n be the number of termsLet an be the nth term.Then, an = d × (n - 1) + a1 A couple of exercises about arithmetic sequences Are the given sequences arithmetic? If so, find the 98th term.a. 2, 6, 9, 11, ...b. -4, 0, 4, 8, 12, ...2, 6, 9, 11, ... is not an arithmetic sequence since the number we add to each term is always the same.-4, 0, 4, 8, 12, ... is an arithmetic sequence since the number we add to each term is always the same.an = d × (n - 1) + a1d = 4n = 98a1 = -4a98 = 4 × (98 - 1) + a1a98 = 4 × (97) + -4a98 = 388 + -4a98 = 384 Everything you may need for a series of rigorous coaching courses for Engineering, Medical and other exams, guided by subject matter experts. One-On-OneTest Seriessee moreInteractive educational solutions, tailored to students' needs to support their learning.Our results reflect we are a great place to pursue your passion and achieve your academic goals.Anirudh Cheriyachanaseri BijayAnirudh Cheriyachanaseri Bijay2.0+croreLIVE Learning10+croreYoutube Views25+croreOn The App57+croreLIVE Classes Piloting the dreams of millions with knowledge and expertise.4.5 crore hours, 10 lakh students in 1000 cities and 50+ countries. PhysicsIIT RoorkeeMathematicsIIT RoorkeeChemistryIIT DelhiMathematicsHOD MathematicsBiologyHOD BiologyMathematicsIIT MadrasGet access to our academic experts from IITs and other top universities, from anywhere and anytime. In algebra, sequences of numbers are valuable for studying what happens as something keeps getting larger or smaller. An arithmetic sequence is defined by the common difference, which is the difference between one number and the next one in the sequence. For arithmetic sequences, this difference is a constant value and can be positive or negative. As a result, an arithmetic sequence keeps getting larger or smaller by a fixed amount every time a new number is added to the list making up the sequence. An arithmetic sequence is a list of numbers in which consecutive terms differ by a constant amount, the common difference. When the common difference is positive, the sequence keeps increasing by a fixed amount, while if it is negative, the sequence decreases. Other common sequences are the geometric sequence, in which terms differ by a common factor, and the Fibonacci sequence, in which each number is the sum of the two previous numbers. An arithmetic sequence is defined by a starting number, a common difference and the number of terms in the sequence. For example, an arithmetic sequence starting with 12, a common difference of 3 and five terms is 12, 15, 18, 21, 24. An example of a decreasing sequence is one starting with the number 3, a common difference of −2 and six terms. This sequence is 3, 1, −1, −3, −5, −7. Arithmetic sequences can also have an infinite number of terms. For example, the first sequence above with an infinite number of terms would be 12, 15, 18, ... and that sequence continues to infinity. An arithmetic sequence has a corresponding series that adds all the terms of the sequence. When the terms are added and the sum is divided by the number of terms, the result is the arithmetic mean or average. The formula for the arithmetic mean is \(\text{mean}= \frac{ \text{sum of }n \text{ terms}}{n}\) A quick way of calculating the mean of an arithmetic sequence is to use the observation that, when the first and last terms are added, the sum is the same as when the second and next to last terms are added or the third and third to last terms. As a result, the sum of the sequence is the sum of the first and last terms times half the number of terms. To get the mean, the sum is divided by the number of terms, so the mean of an arithmetic sequence is half the sum of the first and last terms. For n terms a1 to an, the corresponding formula for the mean m is \(m= \frac{a_1+a_n}{2}\) Infinite arithmetic sequences don't have a last term, and therefore their mean is undefined. Instead, a mean for a partial sum can be found by limiting the sum to a defined number of terms. In that case, the partial sum and its mean can be found the same way as for a non-infinite sequence. Sequences of numbers are often based on observations from experiments or measurements of natural phenomena. Such sequences can be random numbers but often sequences turn out to be arithmetic or other ordered lists of numbers. For example, geometric sequences differ from arithmetic sequences because they have a common factor rather than a common difference. Instead of having a number added or subtracted for each new term, a number is multiplied or divided each time a new term is added. A sequence that is 10, 12, 14, ... as an arithmetic sequence with a common difference of 2 becomes 10, 20, 40, ... as a geometric sequence with a common factor of 2. Other sequences follow completely different rules. For example, the Fibonacci sequence terms are formed by adding the previous two numbers. Its sequence is 1, 1, 2, 3, 5, 8, ... The terms have to be added individually to get a partial sum because the quick method of adding the first and last terms does not work for this sequence. Arithmetic sequences are simple but they have real life applications. If the starting point is known and the common difference can be found, the value of the series at a specific point in the future can be calculated and the average value can be determined as well. Richland: Arithmetic Sequences Markgraf, Bert. "What Is An Arithmetic Sequence?" sciencing.com, . 3 November 2020. APA Markgraf, Bert. (2020, November 3). What Is An Arithmetic Sequence?. sciencing.com. Retrieved from Chicago Markgraf, Bert. What Is An Arithmetic Sequence? last modified August 30, 2022. A sequence made by adding the same value each time. Example: 1, 4, 7, 10, 13, 16, 19, 22, 25, ... In this case each number is 3 larger than the number before it. 3 is the "common difference". home / algebra / sequence / arithmetic sequenceAn arithmetic sequence is a type of sequence in which the difference between each consecutive term in the sequence is constant. For example, the difference between each term in the following sequence is 3: 2, 5, 8, 11, 14, 17, 20... To expand the above arithmetic sequence, starting at the first term, 2, add 3 to determine each consecutive term. This is simple for the first few terms, but using this method to determine terms further along in the sequence gets tedious very quickly. Fortunately, the nth term of an arithmetic sequence can be determined using an = a1 + (n - 1)d where an is the nth term, a1 is the initial term, and d is the constant difference between each term. Using the above sequence, the formula becomes: an = 2 + 3n - 3 = 3n - 1 Therefore, the 100th term of this sequence is: a100 = 3(100) - 1 = 299 This formula allows us to determine the nth term of any arithmetic sequence. Arithmetic sequence vs arithmetic series An arithmetic series is the sum of a finite part of an arithmetic sequence. For example, 2 + 5 + 8 = 15 is an arithmetic series of the first three terms in the sequence above. The sum of a finite arithmetic sequence can be found using the following formula, where n is the number of terms in the sequence, a1 is the first term in the sequence, and an is the nth term, and d is the constant difference between each term. Example Find the sum of the first 7 terms in the arithmetic sequence 1, 7, 13, 19, 25, ... First determine the rest of the first 7 terms. The difference between each consecutive term in the sequence is 6. Since we only need to determine 2 more terms, it is simpler to just add 6 to the last given term and the term after that. The first 7 terms are: 1, 7, 13, 19, 25, 31, 37 The sum of these terms is: The arithmetic sequence is the sequence where the common difference remains constant between any two successive terms. Let us recall what is a sequence. A sequence is a collection of numbers that follow a pattern. For example, the sequence 1, 6, 11, 16, ... is an arithmetic sequence because there is a pattern where each number is obtained by adding 5 to its previous term. We have two arithmetic sequence formulas. The formula for finding nth term of an arithmetic sequence The formula to find the sum of first n terms of an arithmetic sequence If we want to find any term in the arithmetic sequence then we can use the arithmetic sequence formula. Let us learn the definition of an arithmetic sequence and arithmetic sequence formulas along with derivations and a lot more examples for a better understanding. What is an Arithmetic Sequence? An arithmetic sequence is defined in two ways. It is a "sequence where the differences between every two successive terms are the same" (or) In an arithmetic sequence, "every term is obtained by adding a fixed number (positive or negative or zero) to its previous term". The following is an arithmetic sequence as every term is obtained by adding a fixed number 4 to its previous term. Arithmetic Sequence Example Consider the sequence 3, 6, 9, 12, 15, ... is an arithmetic sequence because every term is obtained by adding a constant number (3) to its previous term. Here, The first term, a = 3 The common difference, d = 6 - 3 = 9 - 6 = 12 - 9 = 15 - 12 = ... = 3 Thus, an arithmetic sequence can be written as a, a + d, a + 2d, a + 3d, ... Let us verify this pattern for the above example. a, a + d, a + 2d, a + 3d, a + 4d, ... = 3, 3 + 3, 3 + 2(3), 3 + 3(3), 3 + 4(3),... = 3, 6, 9, 12,15,.... A few more examples of an arithmetic sequence are: 5, 8, 11, 14, ... 80, 75, 70, 65, 60, ... π/2, π, 3π/2, 2π, ... -√2, -2√2, -3√2, -4√2, ... Arithmetic Sequence Formula The first term of an arithmetic sequence is a, its common difference is d, n is the number of terms. The general form of the AP is a, a+d, a+2d, a+3d,.....up to n terms. We have different formulas associated with an arithmetic sequence used to calculate the nth term, the sum of n terms of an AP, or the common difference of a given arithmetic sequence. The arithmetic sequence formula is given as, Nth Term: an = a + (n-1)d Sn = (n/2) [2a + (n - 1)d] d = an - an-1 Nth Term of Ar