I'm not a bot

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The license may not give you all of the permissions necessary for your intended use. For example, other rights such as publicity, privacy, or moral rights may limit how you use the material. In a Geometric Series, every next term is the multiplication of its Previous term by a certain constant, and depending upon the value of the constant, the Series may increase or decrease.Geometric Sequence is given as: a, ar, ar2, ar3, ar4,..... {Infinite Sequence}a, ar, ar2, ar3, ar4, ... arn {Finite Sequence}Geometric Series for the above is written as:a + ar + ar2 + ar3 + ar4 + . . . OR \sum_{k=1}^{\infty}a_k {Infinite Series}a + ar + ar2 + ar3 + ar4 + . . . arn OR \sum_{k=1}^{n}a_k{Finite Series}Where,a = First term, and r = Common Factor.Convergence of Geometric SeriesThe convergence of a geometric series (infinite) depends solely on the value of the common ratio r:Convergent Series: The series converges if the absolute value of the common ratio is less than 1: |r| < 1Divergent Series: The series diverges if the absolute value of the common ratio is equal to or greater than 1: |r| ≥ 1Geometric Series FormulaThe Geometric Series formula for the Finite series is given as,\bold{{S_n =\frac{a(1-r^n)}{1-r}}}WhereSn = sum up to nth term,a = First term, andr = common factor.Derivation for Geometric Series FormulaSuppose a Geometric Series for n terms: Sn = a + ar + ar2 + ar3 + ... + arn-1 . . . (1)Multiplying both sides by the common factor (r):r Sn = ar + ar2 + ar3 + ar4 + ... + arn . . . (2)Subtracting Equation (1) from Equation (2):(r Sn - Sn) = (ar + ar2 + ar3 + ar4 +. . . arn) - (a + ar + ar2 + ar3 + . . . + arn-1)⇒ Sn (r-1) = arn - a⇒ Sn (1 - r) = a (1-rn)⇒ {S_n =\frac{a(1-r^n)}{1-r}}Note: When the value of k starts from 'm', the formula will change.\sum_{k=m}^{n}ar^k=\frac{a(r^m-r^{n+1}}{1-r}, when r≠0For Infinite Geometric Seriesn will tend to Infinity, n → ∞, Putting this in the generalized formula:S_\infty = \sum_{n=1}^{\infty}ar^{n-1} = \frac{a}{1-r}; -1 1[/latex], then the series diverges. Here’s the formula for the infinite geometric series: I just want to reiterate that for the formula to work, the value of the common ratio [latex]r[/latex] must be between [latex]-1[/latex] and [latex]1[/latex], or a fraction with an absolute value less than [latex]1[/latex]. Let’s take a look at some examples! Example 1: Find the sum of the infinite geometric series. [latex]\Large1 + {1 \over 3} + {1 \over 9} + {1 \over {27}} + …[/latex] The first thing we need to do is verify if the sequence is geometric. Divide each term by the preceding term. If the quotient is the same every time we divide, then we have a geometric sequence. As you can see, there’s a common ratio of [latex]\large{1 \over 3}[/latex] which means it is a geometric series. Now that we know we are dealing with a geometric sequence, it’s time for us to figure out if the related series can be summed up to a finite limit. The only test that we have to perform is check if the common ratio [latex]r[/latex] has an absolute value of less than [latex]1[/latex]. Since [latex]\large\left| r \right| = {1 \over 3} < 1[/latex], it implies that the series converges thus we can use the infinite geometric series formula to find the finite sum. From the problem, the first term is [latex]{a_1} = 1[/latex] and and the common ratio is [latex]\large{r = {1 \over 3}}[/latex]. Let’s plug these values in the formula and simplify. Therefore, the sum of the infinite geometric series is [latex]\large{3 \over 2}[/latex]. Example 2: Find the sum of the infinite geometric series. [latex]\large2 + {4 \over 3} + {8 \over 9} + {{16} \over {27}} + …[/latex] First, verify if the series is geometric by dividing each term by the previous one. Does it yield the same quotient? Yes, it does! The common ratio is [latex]\large{r = {2 \over 3}}[/latex]. Since [latex]\large\left| r \right| = {2 \over 3} < 1[/latex], that means the series converges, and therefore we can use the formula to find the finite sum. We know [latex]a_1=2[/latex] and [latex]\large{r = {2 \over 3}}[/latex]. Substitute these values in the formula then simplify. Therefore, the sum of the infinite geometric series is [latex]6[/latex]. Sometimes, an infinite geometric series may be written in a compact form, using the sigma notation [latex]\sum [/latex] that looks like a weird capital letter E. This notation signifies summation in math. As previously stated, an infinite geometric series only converges if the value of the common ratio [latex]r[/latex] is between [latex]-1[/latex] and [latex]1[/latex]. That means, the absolute value of [latex]r[/latex] is less than [latex]1[/latex]. In sigma notation form, the common ratio is usually wrapped around by a parenthesis and being raised to a power. Example 3: Find the sum of the infinite geometric series, if possible. [latex]\large\sum\limits_{n = 1}^\infty {5{{\left({{3 \over 2}} \right)}^{n – 1}}} [/latex] The common ratio is the number inside the parenthesis which is [latex]\large{{3 \over 2}}[/latex]. Since the absolute value of [latex]r[/latex] is NOT less than [latex]1[/latex], that means [latex]\left| r \right| = {3 \over 2} > 1[/latex], this infinite geometric series will not converge which means it will diverge. Therefore, the infinite geometric series won’t have a fixed sum. There’s no need to use the formula. Just a word of caution, if you proceed to use the formula to find the sum, you will still get an answer but your sum is incorrect. Example 4: Find the sum of the infinite geometric series, if possible. [latex]\large\sum\limits_{n = 1}^\infty {8{{\left({ – {3 \over 4}} \right)}^{n – 1}}} [/latex] The absolute value of the common ratio is less than [latex]1[/latex] which means the series converges. [latex]\large\left| r \right| = \left| { – {3 \over 4}} \right| = {3 \over 4} < 1[/latex] Let’s use the formula to find the sum. The first term is [latex]{a_1} = 8[/latex] and the common ratio is [latex]\large{r = – {3 \over 4}}[/latex]. Therefore, the sum of the infinite geometric series is [latex]\large{{32} \over 7}[/latex]. Example 5: Find the sum of the infinite geometric series, if possible. [latex]\large\sum\limits_{n = 1}^\infty {4{{\left({0.3} \right)}^{n – 1}}} [/latex] The absolute value of the common ratio is less than [latex]1[/latex]. It means the series converges and therefore we can find the sum. [latex]\left| r \right| = 0.3 < 1[/latex] The first term is [latex]4[/latex] and the common ratio is [latex]0.3[/latex]. The common ratio can be expressed in fraction form as [latex]\large{3 \over {10}}[/latex]. Therefore, the sum of the infinite geometric series is [latex]\large{{40} \over 7}[/latex] Example 6: Write the repeating decimal as a fraction. [latex]0.7777…[/latex] Notice that we can rewrite this repeating decimal as an infinite series. [latex]0.7777… = 0.7 + 0.07 + 0.007 + …[/latex] Is the infinite series geometric? Let’s divide each term by the previous one and check if we get a common ratio. Yes, the common ratio is [latex]0.1[/latex]. So this is an infinite geometric series. That means we can use the formula to find the finite sum. The first term is [latex]{a_1} = 0.7[/latex] and the common ratio is [latex]r = 0.1[/latex]. Therefore, [latex]0.7777…[/latex] can be written as [latex]\large{7 \over 9}[/latex]. You might also like these tutorials: Tags: Advanced Algebra, Lessons Fact Checked Content Last Updated: 01.12.2023 8 min reading time Content creation process designed by Content cross-checked by Content quality checked by Save Article Save Article Before you can evaluate an infinite geometric series, it helps to know what one is! In order to do that it can be helpful to break it down and first understand what a sequence is. A sequence is a list of numbers that follow a specific rule or pattern. Each number in a sequence is known as a term.There are lots of different types of sequences, including arithmetic and geometric. When thinking about infinite geometric series, it is important to understand what is meant by the term geometric. A geometric sequence is a type of sequence that increases or decreases by a constant multiple. This is known as the common ratio, \(r\). Let's look at some examples!Some examples of geometric sequences include:\(2, 8, 32, 128, 512, \dots\) Here the rule is to multiply by \(4\). Notice that the '\(\dots\)' at the end mean the sequence just keeps following the same pattern forever.\(6, 12, 24, 48, 96\) Here the rule is to multiply by \(2\). \(80, 40, 20, 10, 5\) Here the rule is to multiply by \(\frac{1}{2}\). Now that you understand what us meant by a sequence, you can think about a series.A series is the sum of the terms of a sequence. Let's take a look at some examples.Some examples of series include:\(3+7+11+15+ \dots\) where the original sequence is \(3, 7, 11, 15, \dots\). Again, the '\(\dots\)' means the sum goes on forever, just like the sequence.\(6+12+24+48\) where the original sequence is \(6, 12, 24, 48\).\(70+65+60+55\) where the original sequence is \(70, 65, 60, 55\).Now you can consider each of these definitions to fully understand what an infinite geometric series is. An infinite geometric series is a series that adds up an infinite geometric sequence.Here are some examples. Let's go back to the geometric sequence \(2, 8, 32, 128, 512, \dots\). Find the corresponding geometric series.Answer:First, you can tell this is a geometric sequence because the common ratio here is \(r = 4\), which means that if you divide any two consecutive terms you always get \(4\). You could certainly write down that the geometric series is just adding up all of the terms of the sequence, or\[2 + 8 + 32 + 128 + 512+ \dots\]You could also recognize that there is a pattern here. Each term of the sequence is the previous term multiplied by \(4\). In other words:\[\begin{align} 8 &= 2\cdot 4 \\ 32 &= 8 \cdot 4 = 2 \cdot 4^2 \\ 128 &= 32 \cdot 4 = 2 \cdot 4^3 \\ \vdots \end{align}\]That means you could also write the series as\[2+ 2\cdot 4 + 2\cdot 4^2 + 2\cdot 4^3 + 2 \cdot 4^4 + \dots \]Remember that the common ratio for this series was \(4\), so seeing a multiplication by \(4\) each time makes sense!Infinite geometric series have many real-life applications. Take the population for example. Since the population is rising by a percentage each year, studies can be made to predict how big the population will be in \(5\), \(10\), or even \(50\) years to come by using infinite geometric series. As you saw in the last example, there is a general formula that a geometric series will follow. The general form looks like:\[a +a r+ ar^2+a r^3+\dots\]where the first term of the sequence is \(a\) and \(r\) is the common ratio.Since all geometric series will follow this formula, take time to understand what it means. Let's look at an example of a series in this form. Take the geometric sequence \(6, 12, 24, 48, 96, \dots\) . Find the first term and the common ratio, then write it as a series.Answer:The first term is just the first number in the sequence, so \(a = 6\). You can find the common ratio by dividing any two consecutive terms of the sequence. For example \[\frac{48}{24} = 2\]and\[\frac{24}{2} = 2.\]It doesn't matter which two consecutive terms you divide, you should always get the same ratio. If you don't then it wasn't a geometric sequence to start with! So for this sequence, \(r = 2\).Then using the formula for the geometric series,\[a +a r+ ar^2+a r^3+\dots = 6 + 6\cdot 2 + 6 \cdot 2^2 + 6 \cdot 2^3 + \dots\]This formula can help you to understand exactly what is happening to each term in order to give you the next term. You now now how to find the common ratio for a geometric sequence or series, but other than writing down a formula, what is it good for? The common ratio \(r\) is used to find the next term in a sequence and can have an effect on how the terms increase or decrease. If \(-1 1. Thus the sum does not converge and the series has no sum. Have questions on basic mathematical concepts? Become a problem-solving champ using logic, not rules. Learn the why behind math with our certified experts Book a Free Trial Class Let us now have a look at a few solved examples using the Infinite Series Formula. Example 1: Find the sum of the terms 1/9 + 1/27 + 1/81 + ... to ∞? Solution: To find: Sum of the geometric series Given: a = 1/9, r = 1/3 Using the infinite geometric series formula, Sn = a /(1-r) Sn = (1/9)(1 - 1/3) Sn = 1/6 Answer: The sum of the given terms is 1/6 Example 2: Calculate the sum of series 1/5, 1/10, 1/20, ... if the series contains infinite terms. Solution: To find: Sum of the geometric series Given: a = 1/5, r = 1/5 Using the infinite geometric series formula, Sum = a /(1-r) = (1/5)(1 - 1/5) = 1/4 Answer: The sum of the given terms is 1/4 Example 3: Find the sum of the geometric series 125, 25, 5, 1.....∞ Solution: The series is,125+25+5+1+ ..... a =125, and = 25/125 = 1/5 Using the infinite geometric series formula, Sum = a /(1-r) = 125/(1-1/5) = 125/(4/5) = 625/4 Thus sum to infinity terms is Answer: Thus sum to infinity in the given series 125+25+5+1+ ..... =& 625/4 The infinite geometric series formula is used to find the sum of all the terms in the geometric series without actually calculating them individually. The infinite geometric series formula is given as: \(S_{n}=\dfrac{a}{1-r}\) Where a is the first term r is the common ratio A tangent of a circle in geometry is defined as a straight line that touches the circle at only one point. Can the Sum of an Infinite Geometric Series be Negative? The sum of an infinite series implies that the series is geometric and an infinite arithmetic series can never converge. So if the common ratio is positive there can be no negative sum. What Is the Sum of Infinite GP? The sum to infinite GP means, the sum of terms in an infinite GP. The infinite geometric series formula is S∞ = a/(1 – r), where a is the first term and r is the common ratio. What Is a and r in Infinite Series Formula? In finding the sum of the given infinite geometric series If r 1.\)So, the formula to find the sum of the first \(5\) terms of the geometric series is,\({S_n} = \frac{{a\left({{r^n} – 1} \right)}}{{r – 1}}\)\(\Rightarrow {S_5} = \frac{{10\left({{3^5} – 1} \right)}}{{3 – 1}}\)\(\Rightarrow {S_5} = \frac{{10\left({243 – 1} \right)}}{{3 – 1}}\)\(\Rightarrow {S_5} = \frac{{10\left({242} \right)}}{2}\)\(\Rightarrow {S_5} = \frac{{2420}}{2}\)\(\Rightarrow {S_5} = 1210\)Therefore, the required sum is \(1210.\) Q.6: Find the sum of the first \(7\) terms of the geometric series if \(a = 1,\,r = 2.\)Ans: From the given geometric series, we get, \(a = 1,\,r = 2.\)The formula to find the sum of the first \(7\) terms of the geometric series is,\({S_n} = \frac{{a\left({{r^n} – 1} \right)}}{{r – 1}}\)\(\Rightarrow {S_7} = \frac{{1\left({{2^7} – 1} \right)}}{{2 – 1}}\)\(\Rightarrow {S_7} = \frac{{1\left({128 – 1} \right)}}{1}\)\(\Rightarrow {S_7} = \frac{{1\left({127} \right)}}{1}\)\(\Rightarrow {S_7} = 127\)Therefore, the sum of the first \(7\) terms of the geometric series is \(127.\) Q.7: Find the sum, if it exists for the geometric series: \(10 + 9 + 8 + 7 + ...\)Ans: In this case, we observe that the given series \(10 + 9 + 8 + 7 + ...\) is not a geometric series because the ratio between the consecutive terms is not constant.It is an arithmetic progression. Finding the sum of an infinite arithmetic series is not possible. Q.8: Find the sum of first \(5\) terms of the geometric series: \(1,\,2,\,4,\,8,\,16,\,32,\,....\)Ans: From the given geometric series, we get, \(a = 1,\,r = \frac{2}{1} = 2\)The formula to find the sum of first \(5\) terms of the geometric series is,\({S_n} = \frac{{a\left({{r^n} – 1} \right)}}{{r – 1}}\)So, \({S_5} = \frac{{1\left({{2^5} – 1} \right)}}{{2 – 1}}\)\({S_5} = \frac{{1\left({32 – 1} \right)}}{1}\)\({S_5} = \frac{{32}}{1}\)\({S_5} = 32\)Therefore, the required sum is \(32.\) Below we have provided some of the important practice questions on the sum of geometric series: Find the equivalent fraction of the recurring decimal \(0.797970.....\)Find out the 12th term of the sequence \(3, -6, 9, -12,....?\)Write the first five terms of a GP whose first term is 5 and the common ratio is 6.Find the sum, if it exists for the geometric series: \(20 + 19 + 18 + 17 + ...\)Find the sum of the first \(9\) terms of the geometric series if \(a = 3,\,r = 6.\) The geometric progression is a set of integers generated by multiplying or dividing each preceding term in such a way that there is a common ratio between the terms (that is not equal to \(0\)), and the sum of all these terms is the sum of the geometric progression. The article includes the definition of geometric series, formula to find the sum of n terms of finite and infinite geometric series. We also discussed the derivation of the formula. We mentioned the various applications of the geometric series that helps in understanding its importance. The learning outcome from the topic is that it will help in understanding the method in finding the sum of the different sum of geometric series. The following are the most frequently asked questions on the sum of a Geometric Series: Q.1: Explain the Sum of Geometric Series with an example?Ans: A geometric series is a series where each term is obtained by multiplying or dividing the previous term by a constant number, called the common ratio. And, the sum of the geometric series means the sum of a finite number of terms of the geometric series.Example: Let us consider the series \(27,\,18,\,12,\,...\)Here, we observe that \(\frac{{18}}{{27}} = \frac{2}{3}\) and \(\frac{{12}}{{18}} = \frac{2}{3}.\)So, the ratio of the consecutive terms, in this case, is constant. Hence, the above series can be called a geometric series. Q.2: How do you find the sum of a geometric series?Ans: Let us consider that the first term of a geometric series is \(“a”,\) and the common ratio is \(r\) and the number of terms be \(n.\)To find the sum of a finite geometric series, we use the formula,\({S_n} = \frac{{a\left({{r^n} – 1} \right)}}{{r – 1}}\) for \(r > 1\)\({S_n} = \frac{{a\left({1 – {r^n}} \right)}}{{1 – r}}\) for \(r < 1.\) Q.3: What is the sum of \(n\) terms of a Geometric series?Ans: The formula of the sum of geometric series is given by,\({S_n} = \frac{{a\left({{r^n} – 1} \right)}}{{r – 1}}\) for \(r > 1\)\({S_n} = \frac{{a\left({1 – {r^n}} \right)}}{{1 – r}}\) for \(r < 1\)where \(n \to \)the number of terms, \({a_1} \to \)first term and \(r \to \)common ratio. Q.4: What is the common ratio in geometric series?Ans: The common ratio in the geometric progression is the ratio of any one term in the series divided by the previous term. The letter \(“r”\) is usually used to symbolise it. Q.5: What are the two types of geometric series?Ans: The two types of geometric series are listed below:1. Finite geometric series2. Infinite geometric series Also, refer to some of the helpful articles provided by Embibe: Learn About Application of AP and GP Here We hope you find this article on the Sum of Geometric Series helpful. In case of any queries, you can reach back to us in the comments section, and we will revert with answers. Stay tuned to Embibe for the latest updates.