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 views updated Jun 11 2018Historical development of analytic geometryCartesian coordinate systemDistance between two pointsAlgebraic equations of linesCalculating area using coordinatesEquations for geometric figuresThree-dimensional coordinate systems and beyondResourcesAnalytic geometry is a branch of mathematics that uses algebraic equations to describe the size and position of geometric figures. Developed beginning in the seventeenth century, it is also known as Cartesian geometry or coordinate geometry. The use of a coordinate system to relate geometric points to real numbers is the central idea of analytic geometry. By defining each point with a unique set of real numbers, geometric figures such as lines, circles, and conics can be described with algebraic equations. Analytic geometry has important applications in science and industry alike and is one of the foundations of modern applied mathematics.Historical development of analytic geometryDuring the seventeenth century, finding the solution to problems involving curves became important to industry and science. In astronomy, the slow acceptance of the heliocentric theory (Sun-centered theory) of planetary motion required mathematical formulas that would predict elliptical orbits. Other areas such as optics, navigation, and the military required formulas for things such as determining the curvature of a lens, the shortest route to a destination, and the trajectory of a cannonball. Although the Greeks had developed hundreds of theorems related to curves, these did not provide quantitative values, so they were not useful for practical applications. Consequently, many seventeenth-century mathematicians devoted their attention to the quantitative evaluation of curves. Two French mathematicians, Rene Descartes (1596–1650) and Pierre de Fermat (1601–1665) independently developed the foundations for analytic geometry. Descartes was first to publish his methods in an appendix titled La geometrie of his book Discours de la methode (1637).Cartesian coordinate systemThe link between algebra and geometry was made possible by the development of a coordinate system which allowed geometric ideas, such as point and line, to be described in algebraic terms like real numbers and equations. In the system developed by Descartes, called the rectangular Cartesian coordinate system, points on a geometric plane are associated with an ordered pair of real numbers known as coordinates. Each coordinate describes the location of a single point relative to a fixed point, the origin, which is created by the intersection of a horizontal and a vertical line known as the x-axis and y-axis respectively. The relationship between a point and its coordinates is called one-to-one, since each point corresponds to only one set of coordinates.The x and y axes divide the plane into four quadrants. The sign of the coordinates is either positive or negative, depending in which quadrant the point is located. Starting in the upper right quadrant and working clockwise, a point in the first quadrant would have a positive value for the abscissa and the ordinate. A point in the fourth quadrant (lower right hand corner) would have negative values for each coordinate.The notation P(x, y) describes a point P having coordinates x and y. The x value, called the abscissa, specifies the horizontal distance of a point away from the origin. The y value, known as the ordinate, specifies the vertical distance of a point away from the origin.Distance between two pointsUsing the ideas of analytic geometry, it is possible to calculate the distance between the two points A and B, represented by the line segment AB that connects the points. If two points have the same ordinate but different abscissas, the distance between them is AB = x2 – x1. Similarly, if both points have the same abscissa but different ordinates, the distance is AB = y2 – y1. For points that have neither a common abscissa or ordinate, the Pythagorean theorem is used to determine distance. By drawing horizontal and vertical lines through points A and B to form a right triangle, it can be shown using the distance formula that AB = – (x2 – x1)2 + (y2 – y1)2. The distance between the two points is equal to the length of the line segment AB.In addition to length, it is often desirable to find the coordinates of the midpoint of a line segment. These coordinates can be determined by taking the average of the x and y coordinates of each point. For example, the coordinates for the midpoint M(x, y) between points P(2, 5) and Q(4, 3) are x = (2 + 4)/2 = 3 and y = (5 + 3)/2 = 4.Algebraic equations of linesOne of the most important aspects of analytic geometry is the idea that an algebraic equation can relate to a geometric figure. Consider the equation 2x + 3y = 44. The solution to this equation is an ordered pair (x, y) which represents a point. If the set of every point that makes the equation true (called the locus ) were plotted, the resulting graph would be a straight line. For this reason, equations such as these are known as linear equations. The standard form of a linear equation is Ax + By = C, where A, B, and C are constants and A and B are not both 0. It is interesting to note that an equation such as x = 4 is a linear equation. The graph of this equation is made up of the set of all ordered pairs in which x = 4.The steepness of a line relative to the x-axis can be determined by using the concept of the slope. The slope of a line is defined by the equationThe value of the slope can be used to describe a line geometrically. If the slope is positive, the line is said to be rising. For a negative slope, the line is falling. A slope of zero is a horizontal line, and an undefined slope is a vertical line. If two lines have the same slope, then these lines are parallel.The slope gives us another common form for a linear equation. The slope-intercept form of a linear equation is written y = mx + b, where m is the slope of the line and b is the y intercept. The y intercept is defined as the value for y when x is zero and represents a point on the line that intersects the y axis. Similarly, the x intercept represents a point where the line crosses the x axis and is equal to the value of x when y is zero. Yet another form of a linear equation is the point-slope form, y – y1 = m(x – x1). This form is useful because it allows us to determine the equation for a line if we know the slope and the coordinates of a point.Calculating area using coordinatesOne of the most frequent activities in geometry is determining the area of a polygon such as a triangle or square. By using coordinates to represent the vertices, the areas of any polygon can be determined. The area of triangle OPQ, where O lies at (0, 0), P at (a, b), and Q at (c, d), is found by first calculating the area of the entire rectangle and subtracting the areas of the three right triangles. Thus the area of the triangle formed by points OPQ is = da – (dc/2) – (ab/2) – [(d – b)(a – c)]/2. Through the use of a determinant, it can be shown that the area of this triangle is:This specific case was made easier by the fact that one of the points used for a vertex was the origin.The general equation for the area of a triangle defined by coordinates is represented by the previous equation.In a similar manner, the area for any other polygon can be determined if the coordinates of its points are known.Equations for geometric figuresIn addition to lines and the figures that are made with them, algebraic equations exist for other types of geometric figures. One of the most common examples is the circle. A circle is defined as a figure created by the set of all points in a plane that are a constant distance from a central point. If the center of the circle is at the origin, the formula for the circle is r2 = x2 + y2where r is the distance of each point from the center and called the radius. For example, if a radius of 4 is chosen, a plot of all the x and y pairs that satisfy the equation 42 =x2 + y2 would create a circle. Note, this equation, which is similar to the distance formula, is called the center-radius form of the equation. When the radius of the circle is at the point (a, b) the formula, known as the general form, becomes r2 =(x – a)2 + (y – b)2.The circle is one kind of a broader type of geometric figure known as conic sections. Conic sections are formed by the intersection of a geometric plane and a double-napped cone. After the circle, the most common conics are parabolas, ellipses, and hyperbolas.Curves known as parabolas are found all around us. For example, they are the shape formed by the sagging of telephone wires or the path a ball travels when it is thrown in the air. Mathematically, these figures are described as a curve created by the set of all points in a plane at a constant distance from a fixed point, known as the focus, and a fixed line, called theKEY TERMSAbscissa— The x-coordinate of a point representing its horizontal distance away from the origin.Conic— A geometric figure created by a plane passing through a right circular cone.Coordinate system— A system that relates geometric points to real numbers based on their location in space relative to a fixed point called the origin.Directrix— A line which, together with a focus, determines the shape of a conic section.Ellipse— An eccentric or elongated circle, or oval.Focus— A point, or one of a pair of points, whose position determines the shape of a conic section.Hyperbola— A conic section created by a plane passing through the base of two cones.Intercept— The point at which a curve meets the x or y axes.Linear equations— A mathematical equation which represents a line.Locus— The set of all points that make an equation true.Ordinate— The y-coordinate of a point representing its vertical distance away from the origin.Pythagorean theorem— An idea suggesting that the sum of the squares of the sides of a right triangle is equal to the square of the hypotenuse. It is used to find the distance between two points.Slope— Slope is the ratio of the vertical distance separating any two points on a line to the horizontal distance separating the same two points.directrix. This means that if we take any point on the parabola, the distance of the point from the focus is the same as the distance from the directrix. A line can be drawn through the focus perpendicular to the directrix. This line is called the axis of symmetry of the parabola. The midpoint between the focus and the directrix is the vertex.The equation for a parabola is derived from the distance formula. Consider a parabola that has a vertex at point (h, k). The linear equation for the directrix can be represented by y = k – p, where p is the distance of the focus from the vertex. The standard form of the equation of the parabola is then (x – h)2 =4p(y – k). In this case, the axis of symmetry is a vertical line. In the case of a horizontal axis of symmetry, the equation becomes (y – k)2 = 4p(x – h) where the equation for the directrix is x = h – p. This formula can be expanded to give the more common quadratic formula which is y = Ax2 + Bx + C, such that A does not equal 0.Another widely used conic is an ellipse, which looks like a flattened circle. An ellipse is formed by the graph of the set of points, the sum of whose distances from two fixed points (foci) is constant. To visualize this definition, imagine two tacks placed at the foci. A string is knotted into a circle and placed around the two tacks. The string is pulled taut with a pencil and an ellipse is formed by the path traced. Certain parts of the ellipse are given various names. The two points on an ellipse intersected by a line passing through both foci are called the vertices. The chord connecting both vertices is the major axis and the chord perpendicular to it is known as the minor axis. The point at which the chords meet is known as the center.Again by using the distance formula, the equation for an ellipse can be derived. If the center of the ellipse is at point (h, k) and the major and minor axes have lengths of 2a and 2b respectively, the standard equation isMajor axis is horizontalMajor axis is verticalIf the center of the ellipse is at the origin, the equation simplifies to (x2/a2) + (y2/b2)=1.The “flatness” of an ellipse depends on a number called the eccentricity. This number is given by the ratio of the distance from the center to the focus divided by the distance from the center to the vertex. The greater the eccentricity value, the flatter the ellipse.Another conic section is a hyperbola, which looks like two facing parabolas. Mathematically, it is similar in definition to an ellipse. It is formed by the graph of the set of points, the difference of whose distances from two fixed points (foci) is constant. Notice that in the case of a hyperbola, the difference between the two distances from fixed points is plotted and not the sum of this value, as was done with the ellipse.As with other conics, the hyperbola has various characteristics. It has vertices, the points at which a line passing through the foci intersects the graph, and a center. The line segment connecting the two vertices is called the transverse axis. The simplified equation for a hyperbola with its center at the origin is (x2/a2)– (y2/b2) = 1. In this case, a is the distance between the center and a vertex, b is the difference of the distance between the focus and the center and the vertex and the center.Three-dimensional coordinate systems and beyondGeometric figures such as points, lines, and conics are two-dimensional because they are confined to a single plane. The term two-dimensional is used because each point in this plane is represented by two real numbers. Other geometric shapes like spheres and cubes do not exist in a single plane. These shapes, called surfaces, require a third dimension to describe their location in space. To create this needed dimension, a third axis (traditionally called the z-axis) is added to the coordinate system. Consequently, the location of each point is defined by three real numbers instead of two. For example, a point defined by the coordinates (2, 3, 4) would be located 2 units away from the x axis, 3 units from the y axis, and 4 units from the z axis.The algebraic equations for three-dimensional figures are determined in a way similar to their two-dimensional counterparts. For example, the equation for a sphere is x2 + y2 + z2 = r2. As can be seen, this is slightly more complicated than the equation for its two-dimensional cousin, the circle, because of the additional variable z2.It is interesting to note that just as the creation of a third dimension was possible, more dimensions can be added to our coordinate system. Mathematically, these dimensions can exist, and valid equations have been developed to describe figures in these dimensions. However, it should be noted that this does not mean that these figures physically exist; in fact, at present they only exist in the minds of people who study this type of multidimensional analytic geometry.ResourcesBOOKSBoyer, Carl B. History of Analytic Geometry. New York: Dover Publications, 2004.Larson, Ron. Calculus With Analytic Geometry. Boston: Houghton Mifflin College, 2002.Swokowski, Earl W., and Jeffery A. Cole. Algebra and Trigonometry with Analytic Geometry. 11th ed. Belmont, CA: Brooks Cole, 2006.Perry Romanowski views updated May 29 2018Analytic geometry is a branch of mathematics that uses algebraic equations to describe the size and position of geometric figures on a coordinate system. Developed during the seventeenth century, it is also known as Cartesian geometry or coordinate geometry. The use of a coordinate system to relate geometric points to real numbers is the central idea of analytic geometry. By defining each point with a unique set of real numbers, geometric figures such as lines, circles, and conics can be described with algebraic equations. Analytic geometry has found important applications in science and industry alike.Historical development of analytic geometryDuring the seventeenth century, finding the solution to problems involving curves became important to industry and science. 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For example, they are the shape formed by the sagging of telephone wires or the path a ball travels when it is thrown in the air. Mathematically, these figures are described as a curve created by the set of all points in a plane at a constant distance from a fixed point, known as the focus, and a fixed line, called the directrix. This means that if we take any point on the parabola , the distance of the point from the focus is the same as the distance from the directrix. A line can be drawn through the focus perpendicular to the directrix. This line is called the axis of symmetry of the parabola. The midpoint between the focus and the directrix is the vertex.The equation for a parabola is derived from the distance formula. Consider a parabola that has a vertex at point (h,k). The linear equation for the directrix can be represented by y = k — p, where p is the distance of the focus from the vertex. The standard form of the equation of the parabola is then (x — h)2 = 4p(y — k). In this case, the axis of symmetry is a vertical line. In the case of a horizontal axis of symmetry, the equation becomes (y — k)2 = 4p(x — h) where the equation for the directrix is x = h — p. This formula can be expanded to give the more common quadratic formula which is y = Ax2 + Bx + C, such that A does not equal 0.Another widely used conic is an ellipse , which looks like a flattened circle. An ellipse is formed by the graph of the set of points, the sum of whose distances from two fixed points (foci) is constant. To visualize this definition, imagine two tacks placed at the foci. A string is knotted into a circle and placed around the two tacks. The string is pulled taut with a pencil and an ellipse is formed by the path traced. Certain parts of the ellipse are given various names. The two points on an ellipse intersected by a line passing through both foci are called the vertices. The chord connecting both vertices is the major axis and the chord perpendicular to it is known as the minor axis. The point at which the chords meet is known as the center.Again by using the distance formula, the equation for an ellipse can be derived. If the center of the ellipse is at point (h,k) and the major and minor axes have lengths of 2a and 2b respectively, the standard equation isIf the center of the ellipse is at the origin, the equation simplifies to (x2/a2) + (y2/b2) = 1.The "flatness" of an ellipse depends on a number called the eccentricity. This number is given by the ratio of the distance from the center to the focus divided by the distance from the center to the vertex. The greater the eccentricity value, the flatter the ellipse.Another conic section is a hyperbola , which looks like two facing parabolas. Mathematically, it is similar in definition to an ellipse. 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New York: CRC Press, 1998.Perry RomanowskiKEY TERMS. . . . . .Abscissa—The x-coordinate of a point representing its horizontal distance away from the origin.Conic—A geometric figure created by a plane passing through a right circular cone.Coordinate system—A system that relates geometric point s to real numbers based on their location in space relative to a fixed point called the origin.Directrix—A line which, together with a focus, determines the shape of a conic section.Ellipse—An eccentric or elongated circle, or oval.Focus—A point, or one of a pair of points, whose position determines the shape of a conic section.Hyperbola—A conic section created by a plane passing through the base of two cones.Intercept—The point at which a curve meets the x or y axes.Linear equations—A mathematical equation which represents a line.Locus—The set of all points that make an equation true.Ordinate—The y-coordinate of a point representing its vertical distance away from the origin.Pythagorean theorem—An idea suggesting that the sum of the squares of the sides of a right triangle is equal to the square of the hypotenuse. It is used to find the distance between two points.Slope —Slope is the ratio of the vertical distance separating any two points on a line, to the horizontal distance separating the same two points. views updated May 18 2018quadrant In plane geometry, a quarter of a circle, bounded by radii at right angles to each other and by the arc of the circle. In analytic geometry, it is one of the four sections of a plane divided by an x axis and a y axis. A quadrant is also a device for measuring angles, based on a 90° scale. Taxationincome tax assessment levied upon individual or corporate incomes. Pathologyautism , developmental disability resulting from a neurological disorder that affects the normal functioning of the brain. BuddhismPure Land Buddhism or Amidism, devotional sect of Mahayana Buddhism in China and Japan, centering on worship of the Buddha Amitabha. U.S. HistoryBoston Tea Party 1773. Explorers, Travelers, and Conquerors: BiographiesHernán Cortés or Hernando Cortez , 1485-1547, Spanish conquistador , conqueror of Mexico. French and Benelux Physical GeographyPyrenees , Span. Pirineos, Fr. environmentalism surrogate mother education martial arts Barcelona