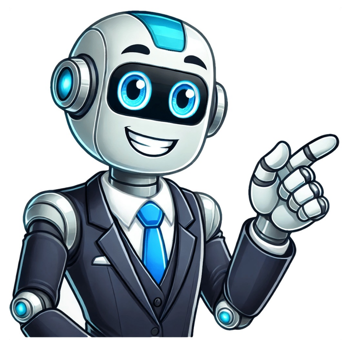

"Completing the Square" is where we take ... For those of you in a hurry, I can tell you that: But if you have time, let me show you how to "Complete the Square" yourself. Say we have a simple expression like x2 + bx. Having x twice in the same expression can make life hard. What can we do? Well, with a little inspiration from Geometry we can convert it, like this: As you can see x2 + bx can be rearranged nearly into a square ... ... and we can complete the square with (b/2)2 In Algebra it looks like this: x2 + bx + (b/2)2 = (x+b/2)2 "Complete the Square" So, by adding (b/2)2 we can complete the square. The result of (x+b/2)2 has x only once, which is easier to use. Keeping the Balance Now ... we can't just add (b/2)2 without also subtracting it too! Otherwise the whole value changes. So let's see how to do it properly with an example: Start with: ("b" is 6 in this case) Complete the Square: (Add and subtract the new term) Simplify it and we are done. The result: x2 + 6x + 7 = (x+3)2 − 2 And now x only appears once, and our job is done! A Shortcut Approach Here is a method you may like, it is quick when you get used to it. First think about the result we want: (x+d)2 + e Which can be expanded to get: x2 + 2dx + d2 + e Now see if we can turn our example into that form to discover d and e. Now we can "force" an answer: We know that 6x must end up as 2dx, so d must be 3 Next we see that 7 must become d2 + e = 9 + e, so e must be −2 And we get the same result (x+3)2 − 2 as above! Now, let us look at a useful application: solving Quadratic Equations ... Solving General Quadratic Equations by Completing the Square We can complete the square to solve a Quadratic Equation (find where it is equal to zero). But a general Quadratic Equation may have a coefficient of a in front of x2: ax2 + bx + c = 0 To deal with that we divide the whole equation by "a" first, then carry on: x2 + (b/a)x + c/a = 0 Steps Now we can solve a Quadratic Equation in 5 steps: Step 1 Divide all terms by a (the coefficient of x2) Step 2 Move the number term (c/a) to the right side of the equation Step 3 Complete the square on the left side of the equation and balance this by adding the same value to the right side of the equation We now have something that looks like (x + p)2 = q, which can be solved this way: Step 4 Take the square root on both sides of the equation Step 5 Subtract the number that remains on the left side of the equation to find x Examples OK, some examples will help! Step 1 can be skipped in this example since the coefficient of x2 is 1 Step 2 Move the number term to the right side of the equation: x2 + 4x = -1 Step 3 Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation. (b/2)2 = (4/2)2 = 22 = 4 x2 + 4x + 4 = -1 + 4 (x + 2)2 = 3 Step 4 Take the square root on both sides of the equation: x + 2 = ±√3 = ±1.73 (to 2 decimals) Step 5 Subtract 2 from both sides: x = ±1.73 – 2 = -3.73 or -0.27 And here is an interesting and useful thing. At the end of step 3 we had the equation: (x + 2)2 = 3 It gives us the vertex (turning point) of x2 + 4x + 1: (-2, -3) Step 1 Divide all terms by 5 x2 – 0.8x – 0.4 = 0 Step 2 Move the number term to the right side of the equation: x2 – 0.8x = 0.4 Step 3 Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation: (b/2)2 = (0.8/2)2 = 0.42 = 0.16 x2 – 0.8x + 0.16 = 0.4 + 0.16 (x – 0.4)2 = 0.56 Step 4 Take the square root on both sides of the equation: x – 0.4 = ±√0.56 = ±0.748 (to 3 decimals) Step 5 Subtract (-0.4) from both sides (in other words, add 0.4): x = ±0.748 + 0.4 = -0.348 or 1.148 Why complete the square when we can just use the Quadratic Formula to solve a Quadratic Equation? Well, one reason is given above, where the new form not only shows us the vertex, but makes it easier to solve. There are also times when the form ax2 + bx + c may be part of a larger question and rearranging it as a(x+d)2 + e makes the solution easier, because x only appears once. For example "x" may itself be a function (like cos(z)) and rearranging it may open up a path to a better solution. Also Completing the Square is the first step in the Derivation of the Quadratic Formula Just think of it as another tool in your mathematics toolbox. 364, 1205, 365, 2331, 2332, 3213, 3896, 3211, 3212, 1206 How did I get the values of d and e from the top of the page? Start with Divide the equation by a Put c/a on other side Add (b/2a)2 to both sides "Complete the Square" Now bring everything back... ... to the left side ... to the original multiple a of x2 And you will notice that we have: a(x+d)2 + e = 0 Where: d = b2a and: e = c − b24a Just like at the top of the page! Copyright © 2025 Rod Pierce Completing the square is a way of rearranging quadratic equations from the general form ax2 + bx + c = 0 to the vertex form a(x – h)2 + k = 0. It is written as a(x + m)2 + n, such that the left side is a perfect square trinomial. Comparing this equation with the general form, we get: ${m=\dfrac{b}{2a}}$${n=c-\dfrac{b^{2}}{4a}}$ It is a form used to solve quadratic equations that is also useful in graphing and analyzing a quadratic function and finding its minimum or maximum values. For example, the quadratic equation x2 – 4x + 4 = 0 can also be written as (x – 2)2. Although the two equations are equivalent, the form (x – 2)2 is sometimes better to work with in some situations. Let us try to understand the concept using the concept of geometry. In a quadratic equation ax2 + bx + c, we will arrange the expression in the form of a perfect square trinomial. Here, x comes twice, which makes it tough to solve. We will simplify the equation further using the concept of geometry as shown below: We will start by dividing all terms by a, the coefficient of x2 Now, let us divide the equation into a square of side x and a rectangle of length ${\dfrac{b}{a}}$ and breadth x as shown: Completing the Square Thus by adding ${\left(\dfrac{b}{2a}\right) ^{2}}$to the equation ${x^{2}+\dfrac{b}{a}x+\dfrac{c}{a}=0}$we can complete the square. Step 1: Dividing all terms by a, the coefficient of x2 ${x^{2}+\dfrac{b}{a}x+\dfrac{c}{a}=0}$ Step 2: Rewriting the equation such that the constant term ‘c’ is the only term on the right side of the equation =>${x^{2}+\dfrac{b}{a}x=-\dfrac{c}{a}}$ Step 3:Adding the square of half the coefficient of the x -term, ${\left(\dfrac{b}{2a}\right) ^{2}}$ to both sides of the equation. Step 4:Factoring the left side as the square of the binomial =>${x^{2}+\dfrac{b}{a}x+\left(\dfrac{b}{2a}\right) ^{2}=-\dfrac{c}{a}+\left(\dfrac{b}{2a}\right) ^{2}}$ = ${\left(x+\dfrac{b}{2a}\right) ^{2}= -\dfrac{c}{a}+\dfrac{b}{(2a}) ^{2}}$ This equation is in the vertex form a(x – h)2 + k. From here, we can find the vertex Step 5: Taking the square root of both sides of the equation =>${\sqrt{\left(x+\dfrac{b}{2a}\right) ^{2}}=\sqrt{-\dfrac{c}{a}+\begin{bmatrix} – \\ 20 \end{bmatrix}^{2}}}$ Step 6: Subtracting the number that remains on the left side of the equation to find x =>${x+\dfrac{b}{2a}-\dfrac{b}{2a}=\sqrt{-\dfrac{c}{a}+\left(\dfrac{b}{2a}\right) ^{2}-}\dfrac{b}{2a}}$ =>${x=\sqrt{-\dfrac{c}{a}+\left(\dfrac{b}{2a}\right) ^{2}-}\dfrac{b}{2a}}$ By completing the square, let us solve the quadratic equation x2 + 4x + 1 = 0. Step 1: Here, this step is not needed as a = 1 Step 2:Moving the constant term to the right side of the equation =>x2 + 4x = -1 Step 3:: Adding the square of half the coefficient of the x -term to both sides of the equation. ${\left(\dfrac{4}{2}\right) ^{2}=\left(2\right) ^{2}=4}$ =>x2 + 4x + 4 = -1 + 4 Step 4:Factoring the left side as the square of the binomial =>(x + 2)2 = 3 This gives us the vertex of the parabola corresponding to the given equation as (-2, -3) Step 5: Taking the square root on both sides of the equation =>x + 2 = ±√3 = ±1.73 Step 6: Subtracting 2 from both sides of the equation => x = (-2 + √3, -2 – √3) Find the value of c in the given quadratic equation x2 + 9x + c that completes the square.Solution:Given the quadratic equation, x2 + 9x + c, here b = 9As we know,x2 + 9x + c=> ${c=\left(\dfrac{b}{2}\right) ^{2}}$=> ${c=\left(\dfrac{9}{2}\right) ^{2}}$=> ${c=\dfrac{81}{4}}$So, the value of c that makes the quadratic equation a perfect square trinomial is ${c=\dfrac{81}{4}}$ Solvex2 + 12x + 32 = 0by the method of completing the squareSolution:x2 + 12x + 32 = 0=>x2 + 12x = -32${\left(\dfrac{12}{2}\right) ^{2}=36}$=>=> x2 + 12x + 36 = -32 + 36=>(x + 6)2 = 4=>${\sqrt{\left(x+6\right) ^{2}}=\sqrt{4}}$=>x + 6 = ±2=> x = (-4, -8) Solve 2x2 – 4x – 5 = 0 by the method of completing the squareSolution:2x2 – 4x – 5 = 0=>${x^{2}-2x-\dfrac{5}{2}=0}$=>${x^{2}-2x=-\dfrac{5}{2}}$${\left(\dfrac{1}{2}\times \left(-2\right) \right) ^{2}=1}$=>${x^{2}-2x+1=\dfrac{5}{2}+1}$=>${\left(x-1\right) ^{2}=\dfrac{7}{2}}$=>${\sqrt{\left(x-1\right) ^{2}}=\pm \sqrt{\dfrac{7}{2}}}$=>${\sqrt{\left(x-1\right) ^{2}}=\pm \sqrt{\dfrac{7}{2}}\times \dfrac{\sqrt{2}}{\sqrt{2}}=\pm \dfrac{\sqrt{14}}{2}}$=>${x-1=\pm \dfrac{\sqrt{14}}{2}}$=>${x=1\pm \dfrac{\sqrt{14}}{2}}$=>${x=\dfrac{2}{2}\pm \dfrac{\sqrt{14}}{2}}$=>${x=\left(\dfrac{2+\sqrt{14}}{2},\dfrac{2-\sqrt{14}}{2}\right)}$ Solving a quadratic equation of the form a(x + m)2 + n, where a = 1 Expanding (x + m)2 + n, we get x2 + 2mx + m2 + n Now, if we compare a quadratic equation of the form ax2 + bx + c with the above equation, we will obtain the value of m and n Let us use the equation x2 + 12x + 32 = 0 Comparing this equation with the equation x2 + 2mx + m2 + n, we get 2mx = 12x =>m = 6 m2 + n = 32 => (6)2 + n = 32 => 36 + n = 32 => n = -4 Thus, we get the same result (x + 6)2 – 4 as obtained while solving the quadratic equation in solved example 1. The rest of the steps remains the same while solving the quadratic equation, and we will get the result x = (-4, -8) Last modified on June 10th, 2024 Completing the square is a method used to solve quadratic equations by rewriting them in a perfect square trinomial format. This technique is especially useful when the quadratic cannot be factored easily or when deriving the quadratic formula.It is a key concept in algebra and calculus, bridging the gap between graphing and algebraic manipulation. Table of Contents What is Completing the Square? Completing the square is a technique to convert a quadratic expression of the form: $ax^2 + bx + c$ into the form: $a(x + d)^2 + e$ This makes it easier to solve equations or graph the parabola. When to Use the Completing the Square Method When quadratic expressions cannot be easily factored To derive the quadratic formula To convert sthe tandard form to vertex form For solving or graphing quadratic equations Formula and Steps To solve $ax^2 + bx + c = 0$ by completing the square: Step 1: Move the constant term to the RHS Step 2: Divide all terms by 'a' (if a ≠ 1) Step 3: Add and subtract $(b/2a)^2$ Step 4: Write the LHS as a perfect square Step 5: Solve by taking square roots Completing the square examples Example 1: Solve $x^2 + 6x + 5 = 0$ by completing the square. Solution: Step 1: $x^2 + 6x = -5$ Step 2: Add $(6/2)^2 = 9$ to both sides $x^2 + 6x + 9 = 4$ Step 3: $(x + 3)^2 = 4$ Step 4: x + 3 = ±2 Answer: x = -1 or x = -5 Example 2: Convert $x^2 - 4x + 1$ into completed square form. Solution: Add and subtract $(4/2)^2 = 4$ $x^2 - 4x + 4 - 4 + 1 = (x - 2)^2 - 3$ What is Completing the Square Method? Completing the square method is a technique that transforms a quadratic equation into a perfect square form. This method helps in solving equations of the form: $ax^2 + bx + c = 0$ By converting the expression into: $(x + p)^2 = q$ We can easily solve for the variable. The method is useful in algebra, calculus, and even physics. Practice Questions Solve: $x^2 + 8x + 7 = 0$ using completing the square Rewrite: $x^2 - 10x + 21$ in completed square form Find the roots of $2x^2 + 12x + 16 = 0$ Solve: $x^2 + 2x = 15$ Convert to vertex form: $x^2 + 4x - 5$ Applications in Real Life Graphing parabolas in physics and economics Optimizing area or revenue functions Solving projectile motion problems Computer graphics and quadratic algorithms Conclusion Completing the square is more than a technique-it's a bridge between algebraic manipulation and geometric understanding. Once mastered, it enhances confidence in solving equations and interpreting quadratic behavior. Related Links : Perfect Square : From square roots to properties - everything you need is here. Discover the full guide on Perfect Squares at Orchids International. Least Square Method : From basics to application - get everything you need to know with examples. Start learning with Orchids International today. Frequently Asked Questions on Completing the Square 1. Why do we use completing the square? Ans: It helps solve quadratic equations and convert them into graphable forms. 2. Is completing the square better than factorization? Ans: Yes, especially when the quadratic isn’t easily factorizable. 3. Can it be used for all quadratic equations? Ans: Yes, any quadratic equation can be solved using this method. 4. Is completing the square part of the quadratic formula derivation? Ans: Yes, the quadratic formula is derived by completing the square. 5. Is it in the Class 10/11/12 curriculum? Ans: Yes, it’s part of the algebra unit in higher classes and foundational for competitive exams. Explore more math concepts at Orchids The International School and boost your confidence in algebra today! home / algebra / solving equations / completing the squareCompleting the square is a method used to solve quadratic equations. It can also be used to convert the general form of a quadratic, ax2 + bx + c to the vertex form a(x - h)2 + k Generally, the goal behind completing the square is to create a perfect square trinomial from a quadratic. A perfect square trinomial is a trinomial that will factor into the square of a binomial. The square of a binomial is a binomial multiplied by itself. Binomials of the form x + n, where n is some constant, are some of the easier binomials to work with. The square of x + n is x2 + 2nx + n2. As you can see, the coefficient of x is 2n. For a quadratic of the form x2 + bx + c, the coefficient of x is already b, so we only need to figure out the value of the constant c. If 2n = b, then n = . Therefore, the binomial used to complete the square is: If we square this binomial, the perfect square trinomial needed to complete the square will equal: or Notice that there are cases where you will subtract . This is because if b is negative, then the constant in the binomial will need to be negative as well. is positive because any number squared is positive. If we go back to the standard form of the quadratic equation, ax2 + bx + c = 0, you will notice two differences between this perfect square trinomial and the quadratic: 1. The trinomial does not contain the coefficient a, or rather, a = 1 2. c is replaced by The objective of completing the square is to satisfy the above two conditions. Let's look at each step in the process: 1. Write the given equation in the standard form of the quadratic equation: Example 2. Factor out and divide both sides by the coefficient of x2 if it does not already equal 1: Note: Dividing both sides by a cancels out a on the left side of the equation. On the right, . 3. Move the constant to the other side of the equation. 4. Add to both sides of the equation (to keep them equivalent): Note: You do not have to simplify on the side with the perfect square trinomial, but it is necessary to on the side with the constant. 5. Factor the side with : 6. Solve for x: Note: Always remember that the square root of a positive number has two answers, one positive and one negative! The method of completing the square is a technique used to rewrite a quadratic expression in a more structured form, making it easier to analyze and solve. The goal is to transform a quadratic polynomial or a quadratic equation into an equivalent expression where the left-hand side is a perfect square trinomial, and the right-hand side is a constant. Given a quadratic polynomial \(p(x) \) of the form: \[p(x) = ax^2 + bx + c \] we aim to rewrite it in the form: \[p(x) = a(x + h)^2 + k \] The previous equation is defined as the vertex form, where \((h, k) \) represents the vertex of the parabola. This form is ideal for identifying properties of the function, such as its minimum or maximum. There is also the form: \[(x + h)^2 = k \] that isolates the squared term, allowing for easy solution by taking square roots. This transformation is systematic and follows a clear set of steps, making it an accessible and reliable method for working with quadratic expressions. We begin with a general quadratic expression \(p(x) = ax^2 + bx + c\). Factor out the leading coefficient \(a \) from the quadratic and linear terms: \[p(x) = a\left(x^2 + \frac{b}{a}x\right) + c \] Now, divide the coefficient of \(x \), which is \(\frac{b}{a} \), by 2, square it, and add and subtract it inside the parentheses: \[p(x) = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 – \left(\frac{b}{2a}\right)^2\right) + c \] Group the first three terms to complete the square: \[p(x) = a\left[\left(x + \frac{b}{2a}\right)^2 – \left(\frac{b}{2a}\right)^2\right] + c \] Distribute \(a \) and simplify: \[p(x) = a\left(x + \frac{b}{2a}\right)^2 – \frac{b^2}{4a} + c \] This expression is of the form: \[p(x) = a(x + h)^2 + k \] where: \[h = \frac{b}{2a} \quad\quad k = c – \frac{b^2}{4a} \] Let’s try to give a geometric perspective to this method by considering the equation: \[x^2 + 6x + 9\] \(x^2 \) represents the area of a square with side \(x \). \(6x \) represents the area of two identical rectangles, each with area \(3x \). \(9 \) is the area of a small square with side \(3 \). The equation can therefore be rewritten as the area of a square with side \(x + 3 \), that is, as \((x + 3)^2 \). Solving the equation then gives the unique solution: \[(x + 3)^2 = 0 \quad \rightarrow \quad x = -3 \] As we can see in examples like this, the solution process is much quicker than applying the quadratic formula. Completing the square offers a more intuitive and often more efficient path, especially when the equation lends itself to neat factoring. However, when the leading coefficient or linear term involves fractions or irrational numbers, this method may become more cumbersome. In such cases, using the quadratic formula might be a more practical alternative. This method can be used to solve a quadratic equation as an alternative to the quadratic formula. While the AC method is useful for factoring quadratics, completing the square provides a systematic approach to rewriting and solving any quadratic equation. Let’s explore how this procedure can be applied to find the roots of the following equation: \[3x^2 – 4x – 1 = 0 \] Isolate the constant term \(c \) on the right side of the equation and divide both sides by \(a \): \[x^2 – \frac{4}{3}x = \frac{1}{3} \] Divide the coefficient \(b \) by 2, square it, and add it to both sides: \[\left(\frac{-4}{6} \right)^2 = \left(\frac{2}{3} \right)^2 = \frac{4}{9} \] Add \(\frac{4}{9} \) to both sides: \[x^2 – \frac{4}{3}x + \frac{4}{9} = \frac{1}{3} + \frac{4}{9} \] The left-hand side is a perfect square trinomial, which can be factored as: \[\left(x – \frac{2}{3}\right)^2 = \frac{1}{3} + \frac{4}{9} \] Simplify the right-hand side: \[\left(x – \frac{2}{3}\right)^2 = \frac{3}{9} + \frac{4}{9} = \frac{7}{9} \] Solve for \(x \) by taking the square root of both sides: \[x – \frac{2}{3} = \pm \sqrt{\frac{7}{9}} = \pm \frac{\sqrt{7}}{3} \] Final solution: \[x = \frac{2}{3} \pm \frac{\sqrt{7}}{3} \] Which simplifies to: \[x = \frac{2 \pm \sqrt{7}}{3} \] In this case, the procedure is more complex and less practical than other methods. For equations like this, using the quadratic formula is generally recommended for efficiency and clarity. The solution to the equation is: \[x = \frac{2 \pm \sqrt{7}}{3} \] Step-by-Step Guide: How to Complete the Square Explained. Figure 01: Are you ready to learn how to find solutions to quadratic equations by completing the square? Completing the square is a method that you can use to solve quadratic equations of the form $ax^2 + bx + c = 0$ (where a, b, and c are all not equal to zero).Note that the equations of the form $ax^2 + bx + c = 0$ are called quadratic equations and they can be rewritten as follows: $ax^2 + bx + c = 0$ $ax^2 + bx = -c$Both of these equations are equivalent to each other, and understanding the relationship between these two equations will help you to understand how to complete the square later on in this guide.In the next section, we will work through three examples of how to complete the square using the following 3-step method:Step #1: Rearrange the quadratic equation so that all of the constants are on one side of the equals sign.Step #2: Add $(b/2)^2$ to both sides of the equal sign.Step #3: Factor and solve.By solving a quadratic equation by completing the square, you are identifying values where the parabola that represents the equation crosses the x-axis.As long as you understand how to follow and apply these three steps, you will be able to solve quadratics by completing the square (provided that they are solvable). Now, let’s gain some experience with using the three step method on how to complete the square by working through some step-by-step practice problems. Figure 02: The solutions to a quadratic equation are the values where the graph crosses the x-axis. Solve: $x^2 - 6x - 16 = 0$For this first example (and all of the practice problems in this guide), we can solve the problem by completing the square using our three step method as follows:Step #1: Rearrange the quadratic equation so that all of the constants are on one side of the equals sign.Let’s start off by noticing that our given quadratic function is indeed in $ax^2 + bx + c = 0$ form, where a=1, b=-6, and c=-16.To complete the first step, we have to move all of the constants (all of the values not attached to variables, to the right side of the equals sign as follows:$x^2 - 6x - 16 = 0$$x^2 - 6x - 16 (+16) = 0 (+16)$$x^2 - 6x = 16$Now we have completed the first step and we are left with a new equivalent equation:Step #2: Add $(b/2)^2$ to both sides of the equal sign.For the next step, we have to find the value of $(b/2)^2$ and add it to both sides of the equals sign.Since we know that b=-6, we can find the value of $(b/2)^2$ by substituting -6 for b as follows:In this case, $(b/2)^2 = 9$, so, to complete Step #2, we simply have to add 9 to both sides of the equal sign as follows:$x^2 - 6x = 16$$x^2 - 6x + 9 = 16 + 9$$x^2 - 6x + 9 = 25$ Figure 03: How to Complete the Square in 3 Easy Steps. Step #3: Factor and solve.Finally, we are ready for the third and final step where we just need to factor and solve.Notice that the left side of the equation of $x^2 - 6x + 9 = 25$ is a trinomial that is factorable as follows:$x^2 - 6x + 9 = (x-3)(x-3)$$x^2 - 6x + 9 = (x-3)^2$In this example, the factors of $x^2 - 6x + 9$ are (x-3)(x-3), which we will express as $(x-3)^2$ since it will allow us to solve the problem as follows:$x^2 - 6x + 9 = 25$$(x-3)^2 = 25$√[$(x-3)^2$] = √[25]x -3 = ± 5In the third step above, we took the square root of both sides of the equation to remove the exponent and we are left with x -3 = ± 5, which means thatWe can now find the solutions to this first example by solving both equations as follows:x - 3 = 5 → x = 8x - 3 = -5 → x = -2All three steps for how to do completing the square are shown in Figure 03 above.Now, we can conclude that the original quadratic equation $x^2 - 6x - 16 = 0$ has two solutions:Final Answer: x = 8 and x = -2This means that the graph of the equation $x^2 - 6x - 16 = 0$ will be a parabola that crosses the x-axis at both (-2,0) and (8,0) as shown in Figure 04 below. Figure 04: How to solve by completing the square: graph explanation Solve: $x^2 + 12x + 32 = 0$We can solve this next example using our 3-step method, just as we did in the previous example, as follows:Step #1: Rearrange the quadratic equation so that all of the constants are on one side of the equals sign.Notice for our given quadratic equation, $x^2 + 12x + 32 = 0$, that a=1, b=12, and c=32.Since our constant c is on the left side of the equation, we simply have to move it to the right side using inverse operations to complete Step #1.$x^2 + 12x + 32 (-32) = 0 (-32)$$x^2 + 12x = -32$After completing the first step, we now have:Step #2: Add $(b/2)^2$ to both sides of the equal sign.Next, we have to add $(b/2)^2$ to both sides of our new equation.In this example, b=12, so we can find the value of $(b/2)^2$ as follows:Since $(b/2)^2 = 36$, we can complete Step #2 by adding 36 to both sides of the equation as follows:$x^2 + 12x = -32$$x^2 + 12x + 36 = -32 + 36$$x^2 + 12x + 36 = 4$ Figure 05: How to complete the square to find the solutions to a quadratic equation. Step #3: Factor and solve.For the final step, we just have to factor and solve for any potential values of x.Just like example #1, we can finish completing the square by factoring the trinomial on the left side of the equation and then solving.In this case, the trinomial on the left side of the equation can be factored as follows:$x^2 + 12x + 36 = (x+6)(x+6)$$x^2 + 12x + 36 = (x+6)^2$Note that whenever you solve a problem using the complete the square method, you will always end up with two identical factors when you complete Step #3.Now that we know that the factors of $x^2 + 12x + 36$ are equal to $(x+6)^2$, we can solve for x as follows:$x^2 + 12x + 36 = 4$$(x+6)^2 = 4$√[$(x+6)^2$] = √[4]x + 6 = ± 2In this case, the original quadratic function $x^2 + 12x + 32 = 0$ will have two solutions:We can determine these two solutions by solving each equation as follows:The entire 3-step method for completing the square for Example #2 is shown in Figure 05 above.Final Answer: x = -4 and x = -8Figure 06 below shows the graph of the parabola represented by $x^2 + 12x + 32$, with x-intercepts at -4 and -8. Figure 06: Solving a quadratic by completing the square helps you to find the x-intercepts of the parabola that represents the equation. Solve: $x^2 + 2x - 7 = 0$Are you starting to get the hang of how to complete the square? Let’s gain some more experience with this next example.Step #1: Rearrange the quadratic equation so that all of the constants are on one side of the equals sign.For this problem, we know that a=1, b=2, and c=-7.For our first step, let’s rearrange the equation so that all of the constants are on the right side:$x^2 + 2x - 7 (+7) = 0 (+7)$$x^2 + 2x = 7$Now we have a new equivalent function:Step #2: Add $(b/2)^2$ to both sides of the equal sign.Now we need to add $(b/2)^2$ to both sides of the equation. Since b=2 in this example, $(b/2)^2$ is equal to:Since $(b/2)^2 = 1$, we can complete the second step by adding 1 to each side of the equation as follows:$x^2 + 2x = 7$$x^2 + 2x + 1 = 7 + 1$$x^2 + 2x + 1 = 8$ Figure 07: How to Complete the Square: Solutions will not always be rational numbers. Step #3: Factor and solve.Now we are ready to factor and solve the equation.We have a trinomial on the left side of the equation that can be factored as follows:$x^2 + 2x + 1 = (x+1)(x+1)$$x^2 + 2x + 1 = (x+1)^2$With these factors in mind, we can solve for x as follows:$x^2 + 2x + 1 = 8$$(x+1)^2 = 8$√[$(x+1)^2$] = √[8]x + 1 = ± √[8]For this third example, the quadratic function $x^2 + 2x - 7 = 0$ will have two solutions:x + 1 = √[8]x + 1 = - √[8]If we continue onto solving these two equations, we will see that, unlike Examples #1 and #2, we do not end up with a rational answer:x + 1 = √[8] → x = -1 + √8x + 1 = - √[8] → x = -1 - √8In cases like this, you can often conclude that:Final Answer: x = -1 + √8 and x = -1 - √8The solutions above are considered exact answers. However, if you are trying to estimate where the parabola will cross the x-axis on the coordinate plane, you could take the problem a step further by approximating for √8 as follows:√8 ≈ 2.83x = -1 + √8 → x = -1 + 2.83 → x =1.83x = -1 + √8 → x = -1 - 2.83 → x =-3.83Now we have two approximate solutions for x:What does this mean? Just like we saw in Examples #1 and #2, the solutions tell you where the graph of the parabola crosses the x-axis. In this example, the graph crosses the x-axis at approximately 1.83 and -3.83, as shown in Figure 08 below. Figure 08: How to complete the square to determine the solutions to a quadratic equation. When learning how to solve quadratic equations of the form $ax^2 + bx + c = 0$, understanding how to complete the square to find the values where x=0 is an important and useful algebra skill that you can use to solve a variety of problems.Whenever you have an equation in $ax^2 + bx + c = 0$ form, you can solve it by following these 3-simple steps to completing the square:Step #1: Rearrange the quadratic equation so that all of the constants are on one side of the equals sign.Step #2: Add $(b/2)^2$ to both sides of the equal sign.Step #3: Factor and solve.Note that the above 3-step method for completing the square can be used to find the solutions of any quadratic equation of the form $ax^2 + bx + c = 0$. These solutions represent the x-values where the parabola that represents the equation crosses the x-axis. Keep Learning: Learn how to find the coordinates of the vertex point of any parabola with this free step-by-step guide. Comment Share — copy and redistribute the material in any medium or format for any purpose, even commercially. Adapt — remix, transform, and build upon the material for any purpose, even commercially. The licensor cannot revoke these freedoms as long as you follow the license terms. Attribution — You must give appropriate credit , provide a link to the license, and indicate if changes were made . You may do so in any reasonable manner, but not in any way that suggests the licensor endorses you or your use. ShareAlike — If you remix, transform, or build upon the material, you must distribute your contributions under the same license as the original. No additional restrictions — You may not apply legal terms or technological measures that legally restrict others from doing anything the license permits. You do not have to comply with the license for elements of the material in the public domain or where your use is permitted by an applicable exception or limitation . No warranties are given. The license may not give you all of the permissions necessary for your intended use. For example, other rights such as publicity, privacy, or moral rights may limit how you use the material. "Completing the Square" is where we take ... For those of you in a hurry, I can tell you that: But if you have time, let me show you how to "Complete the Square" yourself. Say we have a simple expression like x2 + bx. Having x twice in the same expression can make life hard. What can we do? Well, with a little inspiration from Geometry we can convert it, like this: As you can see x2 + bx can be rearranged nearly into a square ... ... and we can complete the square with (b/2)2 In Algebra it looks like this: x2 + bx + (b/2)2 = (x+b/2)2 "Complete the Square" So, by adding (b/2)2 we can complete the square. The result of (x+b/2)2 has x only once, which is easier to use. Keeping the Balance Now ... we can't just add (b/2)2 without also subtracting it too! Otherwise the whole value changes. So let's see how to do it properly with an example: Start with: ("b" is 6 in this case) Complete the Square: (Add and subtract the new term) Simplify it and we are done. The result: x2 + 6x + 7 = (x+3)2 − 2 And now x only appears once, and our job is done! A Shortcut Approach Here is a method you may like, it is quick when you get used to it. First think about the result we want: (x+d)2 + e Which can be expanded to get: x2 + 2dx + d2 + e Now see if we can turn our example into that form to discover d and e. Now we can "force" an answer: We know that 6x must end up as 2dx, so d must be 3 Next we see that 7 must become d2 + e = 9 + e, so e must be −2 And we get the same result (x+3)2 − 2 as above! Now, let us look at a useful application: solving Quadratic Equations ... Solving General Quadratic Equations by Completing the Square We can complete the square to solve a Quadratic Equation (find where it is equal to zero). But a general Quadratic Equation may have a coefficient of a in front of x2: ax2 + bx + c = 0 To deal with that we divide the whole equation by "a" first, then carry on: x2 + (b/a)x + c/a = 0 Steps Now we can solve a Quadratic Equation in 5 steps: Step 1 Divide all terms by a (the coefficient of x2) Step 2 Move the number term (c/a) to the right side of the equation Step 3 Complete the square on the left side of the equation and balance this by adding the same value to the right side of the equation We now have something that looks like (x + p)2 = q, which can be solved this way: Step 4 Take the square root on both sides of the equation Step 5 Subtract the number that remains on the left side of the equation to find x Examples OK, some examples will help! Step 1 can be skipped in this example since the coefficient of x2 is 1 Step 2 Move the number term to the right side of the equation: x2 + 4x = -1 Step 3 Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation. (b/2)2 = (4/2)2 = 22 = 4 x2 + 4x + 4 = -1 + 4 (x + 2)2 = 3 Step 4 Take the square root on both sides of the equation: x + 2 = ±√3 = ±1.73 (to 2 decimals) Step 5 Subtract 2 from both sides: x = ±1.73 – 2 = -3.73 or -0.27 And here is an interesting and useful thing. At the end of step 3 we had the equation: (x + 2)2 = 3 It gives us the vertex (turning point) of x2 + 4x + 1: (-2, -3) Step 1 Divide all terms by 5 x2 – 0.8x – 0.4 = 0 Step 2 Move the number term to the right side of the equation: x2 – 0.8x = 0.4 Step 3 Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation: (b/2)2 = (0.8/2)2 = 0.42 = 0.16 x2 – 0.8x + 0.16 = 0.4 + 0.16 (x – 0.4)2 = 0.56 Step 4 Take the square root on both sides of the equation: x – 0.4 = ±√0.56 = ±0.748 (to 3 decimals) Step 5 Subtract (-0.4) from both sides (in other words, add 0.4): x = ±0.748 + 0.4 = -0.348 or 1.148 Why complete the square when we can just use the Quadratic Formula to solve a Quadratic Equation? Well, one reason is given above, where the new form not only shows us the vertex, but makes it easier to solve. There are also times when the form ax2 + bx + c may be part of a larger question and rearranging it as a(x+d)2 + e makes the solution easier, because x only appears once. For example "x" may itself be a function (like cos(z)) and rearranging it may open up a path to a better solution. Also Completing the Square is the first step in the Derivation of the Quadratic Formula Just think of it as another tool in your mathematics toolbox. 364, 1205, 365, 2331, 2332, 3213, 3896, 3211, 3212, 1206 How did I get the values of d and e from the top of the page? Start with Divide the equation by a Put c/a on other side Add (b/2a)2 to both sides "Complete the Square" Now bring everything back... ... to the left side ... to the original multiple a of x2 And you will notice that we have: a(x+d)2 + e = 0 Where: d = b2a and: e = c − b24a Just like at the top of the page! Copyright © 2025 Rod Pierce Download Article A walkthrough for the entire process of completing the square Download Article Completing the square is a helpful technique that allows you to rearrange a quadratic equation into a neat form that makes it easier to visualize or even solve. It’s used to determine the vertex of a parabola and to find the roots of a quadratic equation. If you’re just starting out with completing the square, or if the math isn’t exactly adding up, follow along with these easy steps to become a quadratic whiz. To complete the square with x 2 + a x + C = y {\displaystyle x^{2}+ax+C=y} , subtract C from both sides, then add (1 2 a) 2 {\displaystyle \left({\frac {1}{2}}a\right)^{2}} to both sides. Now the equation can be factored as (x + 1 2 a) 2 = y − C + 1 2 a {\displaystyle \left(x+{\frac {1}{2}}a\right)^{2}=y-C+{\frac {1}{2}}a} , where the left side is a perfect square. 1 Write down the equation. Let's say you're working with the following equation: x 2 + 10 x + 5 = 0. {\displaystyle x^{2}+10x+5=0.} 2 Subtract the last term from both sides of the equation. To isolate your x terms, move the constant (number without an “x”) to the other side of the equation by subtracting it from both sides. x 2 + 10 x = − 5. {\displaystyle x^{2}+10x=-5.} [1] Advertisement 3 Halve the coefficient of the x term and square it. Take the number in front of the x (second) term in the equation, divide it by 2, and then square it.[2] In this example, the second term is 10 x , {\displaystyle 10x,} so 10 2 = 5 , {\displaystyle {\frac {10}{2}}=5,} and 5 2 = 25 {\displaystyle 5^{2}=25} . 4 Add the term to both sides of the equation. By adding this term to the left side of the equation, you create a polynomial that is a perfect square. To keep the equation balanced, add the number to the right side of the equation, as well.[3] x 2 + 10 x + 25 = − 5 + 25. {\displaystyle x^{2}+10x+25=-5+25.} 5 Factor the left side of the equation. Now that the left side is a perfect square, you can rewrite it as (x + the halved second term)2, or (x + 5) 2 {\displaystyle \left(x+5\right)^{2}} for this example. You can check your math by multiplying it out and seeing if it gives you the first three terms from the last step.[4] 6 Take the square root of both sides. To solve your equation, undo the square on the left by taking the square root of both sides.[5] (x + 5) 2 = ± 20 {\displaystyle {\sqrt {\left(x+5\right)^{2}}}=\pm {\sqrt {20}}} . 7 Solve to find the roots of the equation. When you take the square root of the right side of the equation, the result is both negative and positive, since negative numbers are also positive when they’re squared. To solve this equation, break it into two parts: x = − 5 + 20 {\displaystyle x=-5+{\sqrt {20}}} and x = − 5 − 20 {\displaystyle x=-5-{\sqrt {20}}} .[6] Advertisement 1 Write your equation. For this example, try an equation where the x 2 {\displaystyle x^{2}} term has a coefficient, like 3 x 2 + 4 x − 15 = y {\displaystyle 3x^{2}+4x-15=y} . Vertex form is a way of writing a quadratic equation (a x 2 + b x + C) {\displaystyle \left(ax^{2}+bx+C\right)} with the coordinates of the vertex of the parabola formed by that equation. Vertex form is written as f (x) = a (x − h) 2 + k {\displaystyle f(x)=a\left(x-h\right)^{2}+k} where (h , k) {\displaystyle (h,k)} is the vertex of a parabola.[7] 2 Move the constant to the right side of the equation. Isolate the x terms by adding 15 to both sides of the equation.[8] 3 x 2 + 4 x − 15 + 15 = y + 15 {\displaystyle 3x^{2}+4x-15+15=y+15} 3 Factor out the coefficient of the squared term from the first 2 terms. To complete the square, the leading coefficient has to be 1, so factor 3 out of the left side of the equation.[9] 3 (x 2 + 4 3 x) = y + 15 {\displaystyle 3\left(x^{2}+{\frac {4}{3}}x\right)=y+15} . 4 Halve the second term and square it. The second term, also known as the b term, in the equation, is 4 3 {\displaystyle {\frac {4}{3}}} . First, divide it by two: 4 3 ∗ 1 2 = 4 6 = 2 3 {\displaystyle {\frac {4}{3}}*{\frac {1}{2}}={\frac {4}{6}}={\frac {2}{3}}} . Then, square the term: 2 2 3 2 = 4 9 {\displaystyle {\frac {2^{2}}{3^{2}}}={\frac {4}{9}}} .[10] Completing the square refers to finding a constant “C” that you can add to the first two terms to make a perfect square trinomial, which can be factored into the expression (x + a) 2 {\displaystyle \left(x+a\right)^{2}} . The number you get at this step is the constant that completes the square. 5 Add the term to both sides of the equation. Use the new term to make a perfect square trinomial by adding it into the parentheses.[11] Since you factored out a 3, multiply the new term by three before adding it to the right side of the equation with the y term.[12] 3 (x 2 + 4 3 x + 4 9) = y + 15 + 3 (4 9) 3 (x 2 + 4 3 x + 4 9) = y + 15 + 4 3 3 (x 2 + 4 3 x + 4 9) = y + 49 3 {\displaystyle {\begin{aligned}3\left(x^{2}+{\frac {4}{3}}x+{\frac {4}{9}}\right)&=y+15+3\left({\frac {4}{9}}\right)\\3\left(x^{2}+{\frac {4}{3}}x+{\frac {4}{9}}\right)&=y+15+{\frac {4}{3}}\\3\left(x^{2}+{\frac {4}{3}}x+{\frac {4}{9}}\right)&=y+{\frac {49}{3}}\end{aligned}}} 6 Factor the trinomial as a square and isolate “y.” Now that you have a perfect square in the parenthesis, factor it out using the halved b term from before, in this case 2 3 {\displaystyle {\frac {2}{3}}} . Write your equation as a factored square, subtract the constant from the y side, and add it to the x side.[13] 3 (x + 2 3) 2 − 49 3 = y + 49 3 − 49 3 {\displaystyle 3\left(x+{\frac {2}{3}}\right)^{2}-{\frac {49}{3}}=y+{\frac {49}{3}}-{\frac {49}{3}}} 3 (x + 2 3) 2 − 49 3 = y {\displaystyle 3\left(x+{\frac {2}{3}}\right)^{2}-{\frac {49}{3}}=y} If you’re using the vertex form f (x) = a (x − h) 2 + k {\displaystyle f(x)=a(x-h)^{2}+k} to find the vertex of a parabola (h, k), remember that “h” has to be negative and “k” has to be positive. In this example, the vertex is (− 2 3 , − 49 3) {\displaystyle \left(-{\frac {2}{3}},-{\frac {49}{3}}\right)} .[14] Advertisement 1 Write down the problem. Let's say you're working with the following equation: 3 x 2 + 4 x + 5 = 6 {\displaystyle 3x^{2}+4x+5=6} . 2 Move the constant terms to the left side of the equation. The constant terms are any terms that aren't attached to a variable.[15] In this case, you have 5 on the left side and 6 on the right side, so subtract 6 from both sides of the equation. Now you have 3 x 2 + 4 x − 1 = 0. {\displaystyle 3x^{2}+4x-1=0.} [16] 3 Factor out the coefficient of the squared term. In this case, 3 is the coefficient of the x 2 {\displaystyle x^{2}} term. Divide each term by 3, then put the terms into parenthesis with a 3 in front. So, 3 x 2 ÷ 3 = x 2 {\displaystyle 3x^{2}\div 3=x^{2}} , 4 x ÷ 3 = 4 3 x {\displaystyle 4x\div 3={\frac {4}{3}}x} , and 1 ÷ 3 = 1 3 {\displaystyle 1\div 3={\frac {1}{3}}} . Now the equation is: 3 (x 2 + 4 3 x − 1 3) = 0. {\displaystyle 3\left(x^{2}+{\frac {4}{3}}x-{\frac {1}{3}}\right)=0.} [17] 4 Divide by the constant you just factored out. Since you divided each term by 3, it can be removed without impacting the equation. Now you have x 2 + 4 3 x − 1 3 = 0. {\displaystyle x^{2}+{\frac {4}{3}}x-{\frac {1}{3}}=0.} [18] 5 Move the constant to the right side of the equation. Isolate the x terms by moving − 1 3 {\displaystyle -{\frac {1}{3}}} to the other side of the equal sign.[19] x 2 + 4 3 x − 1 3 + 1 3 = 0 + 1 3 {\displaystyle x^{2}+{\frac {4}{3}}x-{\frac {1}{3}}+{\frac {1}{3}}=0+{\frac {1}{3}}} 6 Halve the second term and square it. Next, take the second term, 4 3 {\displaystyle {\frac {4}{3}}} , also known as the b term, and halve and square it to complete the square. When you’re done, add it to both sides of the equation to keep it balanced.[20] x 2 + 4 3 x + 4 9 = 1 3 + 4 9 {\displaystyle x^{2}+{\frac {4}{3}}x+{\frac {4}{9}}={\frac {1}{3}}+{\frac {4}{9}}} 4 3 ∗ 1 2 = 2 3 {\displaystyle {\frac {4}{3}}*{\frac {1}{2}}={\frac {2}{3}}} 2 2 3 2 = 4 9 {\displaystyle {\frac {2^{2}}{3^{2}}}={\frac {4}{9}}} 7 Write the left side of the equation as a perfect square. Since you've already used a formula to find the missing term, all you have to do is put x and half of the second coefficient in parentheses and square them, like so: (x + 2 3) 2 {\displaystyle \left(x+{\frac {2}{3}}\right)^{2}} . The equation should now read: (x + 2 3) 2 = 7 9 {\displaystyle \left(x+{\frac {2}{3}}\right)^{2}={\frac {7}{9}}} .[21] Note that factoring that perfect square will give you the three terms: x 2 + 4 3 x + 4 9 {\displaystyle x^{2}+{\frac {4}{3}}x+{\frac {4}{9}}} . 8 Take the square root of both sides. On the left side of the equation, the square root of (x + 2 3) 2 {\displaystyle \left(x+{\frac {2}{3}}\right)^{2}} is just x + 2 3 {\displaystyle x+{\frac {2}{3}}} . On the right side, the square root of the denominator, 9, is 3, and the square root of 7 is 7 {\displaystyle {\sqrt {7}}} , so the square root is ± 7 3 {\displaystyle \pm {\frac {\sqrt {7}}{3}}} .[22] Remember to write ± {\displaystyle \pm } because a square root can be positive or negative. 9 Isolate the variable. To isolate the variable x, move the constant term 2 3 {\displaystyle {\frac {2}{3}}} over to the right side of the equation. You now have two possible answers for x: ± 7 3 − 2 3 {\displaystyle \pm {\frac {\sqrt {7}}{3}}-{\frac {2}{3}}} . You can leave it at that or find the actual square root of 7 if you need an answer without the radical sign.[23] Advertisement Add New Question Question Why do you halve the b value and then square it? It makes no sense to me. It does seem strange and arbitrary, but there is a reason for it. The power move is taking the square root of both sides, but you can't simplify the square root of most polynomials. The step you ask about is a setup move to make the power move work. If I have, for example, x^2 + 4x = 5, and take the square root of both sides, nothing happens, it just makes a mess. But if I add 4 to both sides first and take the square root of both sides of x^2 + 4x + 4 = 9, it simplifies to |x+2| = 3 and the quadratic equation is reduced to a linear equation. Question What's the completing the square formula if x > 1? The value of x doesn't matter. The process remains as shown above. Question In Part 1 of 2, how did you get 11/9 in Step 8? Both sides of that equation are being divided by 3 (to get rid of the coefficient of the first term). Dividing the second term (11/3) by 3 gives us 11/9. See more answers Ask a Question Advertisement Thanks Thanks Advertisement Thanks for reading our article! If you’d like to learn more about math, check out our in-depth interview with David Jia. This article was co-authored by David Jia and by wikiHow staff writer, Carmine Shannon. David Jia is an Academic Tutor and the Founder of LA Math Tutoring, a private tutoring company based in Los Angeles, California. With over 10 years of teaching experience, David works with students of all ages and grades in various subjects, as well as college admissions counseling and test preparation for the SAT, ACT, ISEE, and more. After attaining a perfect 800 math score and a 690 English score on the SAT, David was awarded the Dickinson Scholarship from the University of Miami, where he graduated with a Bachelor’s degree in Business Administration. Additionally, David has worked as an instructor for online videos for textbook companies such as Larson Texts, Big Ideas Learning, and Big Ideas Math. This article has been viewed 475,711 times. Co-authors: 32 Updated: July 7, 2025 Views: 475,711 Categories: Algebra Print Send fan mail to authors Thanks to all authors for creating a page that has been read 475,711 times. "I tutor math to high school students. Since everyone learns a little differently, I am always looking for different ways to present material. The second method in this article is an example of presenting the idea a little differently!"..." more Share your story Home>Completing the Square – Explanation & ExamplesExample 3Solve x2 + 10x − 4 = 0SolutionRewrite the quadratic equation by isolating c on the right side.x2 + 10x = 4Add both sides of the equation by (10/2)2 = 52 = 25.= x2 + 10x + 25 = 4 + 25= x2 + 10x + 25 = 29Write the left side as a square(x + 5) 2 = 29x = -5 ±√29x = 0.3852, – 10.3852Example 4Solve 3x2 – 5x + 2 = 0SolutionDivide each term of the equation by 3 to make the leading coefficient equals to 1.x2 – 5/3 x + 2/3 = 0Comparing with the standard form; (x + b/2)2 = -(c-b2/4)b = -5/3; c = 2/3c – b2/4 = 2/3 – [(5/3)2/4] = 2/3 – 25/36 = -1/36Therefore,⇒ (x – 5/6)2 = 1/36⇒ (x – 5/6)= ± √(1/36)⇒ x – 5/6 = ±1/6⇒ x = 1, -2/3Example 5Solve x2 – 6x – 3 = 0Solutionx2 – 6x = 3x2 – 6x + (-3)2 = 3 + 9(x – 3)2 = 12x – 3= ± √12x = 3 ± 2√3Example 6Solve: 7x2 − 8x + 3=0Solution7x2 − 8x = −3x2 −8x/7 = −3/7x2 - 8x/7 +(−4/7)2 = −3/7+16/49(x − 4/7)2 = −5/49x = 4/7 ± (√7) i/5(x – 3)2 = 12x – 3 = ±√12x = 3 ± 2√3Example 7Solve 2x2 – 5x + 2 = 0SolutionDivide each term by 2x2 – 5x/2 + 1 = 0⇒ x2 – 5x/2= -1Add (1/2 × −5/2) = 25/16 to both sides of the equation.= x2 – 5x/2 + 25/16 = -1 + 25/16= (x – 5/4)2 = 9/16= (x – 5/4)2 = (3/4)2= x – 5/4= ± 3/4⇒ x = 5/4 ± 3/4x = 1/2, 2Example 8Solve x2- 10x -11= 0SolutionWrite the trinomial as a perfect square(x2 – 10x + 25) – 25 – 11 = 36⇒ (x – 5)2 – 36 =0⇒ (x – 5)2 = 36Find the square roots on both sides of the equationx – 5 = ± √36x -5 = ±6x = −1 or x =11Example 9Solve the following equation by completing the squarex2 + 10x – 2 = 0Solutionx2 + 10x – 2 = 0⇒ x2 + 10x = 2⇒ x2 + 10x + 25 = 2 + 25⇒ (x + 5)2 = 27Find the square roots on both sides of the equation⇒ x + 5 = ± √27⇒ x + 5 = ± 3√3x = -5 ± 3√3Example 10Solve x2 + 4x + 3 = 0Solutionx2 + 4x + 3 = 0 ⇒ x2 + 4x = -3x2 + 4x + 4 = – 3 + 4Write the trinomial as a perfect square(x + 2)2 = 1Determine the square roots on both sides.(x + 2) = ± √1x= -2+1= -1ORx = -2-1= -3Example 11Solve the equation below using the method of completing the square.2x2 – 5x + 1 = 0Solutionx2−5x/2 + 1/2=0x2 −5x/2 = −1/2(1/2) (−5/2) =−5/4(−5/4)2 = 25/16x2 – 5x/2 + 25/16 = −1/2 + 25/16(x – 5/4) 2 = 17/16Find the square of both sides.(x – 5/4) = ± √ (17/16)x = [5 ± √ (17)]/4 "Completing the Square" is where we take ... For those of you in a hurry, I can tell you that: But if you have time, let me show you how to "Complete the Square" yourself. Say we have a simple expression like x2 + bx. Having x twice in the same expression can make life hard. What can we do? Well, with a little inspiration from Geometry we can convert it, like this: As you can see x2 + bx can be rearranged nearly into a square ... ... and we can complete the square with (b/2)2 In Algebra it looks like this: x2 + bx + (b/2)2 = (x+b/2)2 "Complete the Square" So, by adding (b/2)2 we can complete the square. The result of (x+b/2)2 has x only once, which is easier to use. Keeping the Balance Now ... we can't just add (b/2)2 without also subtracting it too! Otherwise the whole value changes. So let's see how to do it properly with an example: Start with: ("b" is 6 in this case) Complete the Square: (Add and subtract the new term) Simplify it and we are done. The result: x2 + 6x + 7 = (x+3)2 − 2 And now x only appears once, and our job is done! A Shortcut Approach Here is a method you may like, it is quick when you get used to it. First think about the result we want: (x+d)2 + e Which can be expanded to get: x2 + 2dx + d2 + e Now see if we can turn our example into that form to discover d and e. Now we can "force" an answer: We know that 6x must end up as 2dx, so d must be 3 Next we see that 7 must become d2 + e = 9 + e, so e must be −2 And we get the same result (x+3)2 − 2 as above! Now, let us look at a useful application: solving Quadratic Equations ... Solving General Quadratic Equations by Completing the Square We can complete the square to solve a Quadratic Equation (find where it is equal to zero). But a general Quadratic Equation may have a coefficient of a in front of x2: ax2 + bx + c = 0 To deal with that we divide the whole equation by "a" first, then carry on: x2 + (b/a)x + c/a = 0 Steps Now we can solve a Quadratic Equation in 5 steps: Step 1 Divide all terms by a (the coefficient of x2) Step 2 Move the number term (c/a) to the right side of the equation Step 3 Complete the square on the left side of the equation and balance this by adding the same value to the right side of the equation We now have something that looks like (x + p)2 = q, which can be solved this way: Step 4 Take the square root on both sides of the equation Step 5 Subtract the number that remains on the left side of the equation to find x Examples OK, some examples will help! Step 1 can be skipped in this example since the coefficient of x2 is 1 Step 2 Move the number term to the right side of the equation: x2 + 4x = -1 Step 3 Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation. (b/2)2 = (4/2)2 = 22 = 4 x2 + 4x + 4 = -1 + 4 (x + 2)2 = 3 Step 4 Take the square root on both sides of the equation: x + 2 = ±√3 = ±1.73 (to 2 decimals) Step 5 Subtract 2 from both sides: x = ±1.73 – 2 = -3.73 or -0.27 And here is an interesting and useful thing. At the end of step 3 we had the equation: (x + 2)2 = 3 It gives us the vertex (turning point) of x2 + 4x + 1: (-2, -3) Step 1 Divide all terms by 5 x2 – 0.8x – 0.4 = 0 Step 2 Move the number term to the right side of the equation: x2 – 0.8x = 0.4 Step 3 Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation: (b/2)2 = (0.8/2)2 = 0.42 = 0.16 x2 – 0.8x + 0.16 = 0.4 + 0.16 (x – 0.4)2 = 0.56 Step 4 Take the square root on both sides of the equation: x – 0.4 = ±√0.56 = ±0.748 (to 3 decimals) Step 5 Subtract (-0.4) from both sides (in other words, add 0.4): x = ±0.748 + 0.4 = -0.348 or 1.148 Why complete the square when we can just use the Quadratic Formula to solve a Quadratic Equation? Well, one reason is given above, where the new form not only shows us the vertex, but makes it easier to solve. There are also times when the form ax2 + bx + c may be part of a larger question and rearranging it as a(x+d)2 + e makes the solution easier, because x only appears once. For example "x" may itself be a function (like cos(z)) and rearranging it may open up a path to a better solution. Also Completing the Square is the first step in the Derivation of the Quadratic Formula Just think of it as another tool in your mathematics toolbox. 364, 1205, 365, 2331, 2332, 3213, 3896, 3211, 3212, 1206 How did I get the values of d and e from the top of the page? Start with Divide the equation by a Put c/a on other side Add (b/2a)2 to both sides "Complete the Square" Now bring everything back... ... to the left side ... to the original multiple a of x2 And you will notice that we have: a(x+d)2 + e = 0 Where: d = b2a and: e = c − b24a Just like at the top of the page! Copyright © 2025 Rod Pierce Completing the square is a method that is used for converting a quadratic expression of the form ax2 + bx + c to the vertex form a(x - h)2 + k. The most common application of completing the square is in solving a quadratic equation. This can be done by rearranging the expression obtained after completing the square: a(x + m)2 + n, such that the left side is a perfect square trinomial. Let us learn more about completing the square formula, its method and the process of completing the square step-wise. We will discuss its applications using solved examples for a better understanding. What is Completing the Square? Completing the square is a method in algebra that is used to write a quadratic expression in a way such that it contains the perfect square. In simple words, we can say that completing the square is a process where consider a quadratic equation of the ax2 + bx + c = 0 and change it to write it in perfecting the square form a(x + p)2 + q = 0. Completing the square method is useful in: Completing the square method is usually introduced in class 10. Check the following links that you may find helpful. Quadratic Equations Formulas Class 10 Quadratic Equation Calculator Completing the Square Method The most common application of completing the square method is factorizing a quadratic equation, and henceforth finding the roots or zeros of a quadratic polynomial or a quadratic equation. We know that a quadratic equation of the form ax2 + bx + c = 0 can be solved by the factorization method. But sometimes, factorizing the quadratic expression ax2 + bx + c is complex or NOT possible. Let us have a look at the following example to understand this case. For example: x2 + 2x + 3 cannot be factorized as we cannot find two numbers whose sum is 2 and whose product is 3. In such cases, we write it in the form a(x + m)2 + n by completing the square. Since we have (x + m) whole squared, we say that we have "completed the square" here. But, how do we complete the square? Let us understand the concept in detail in the following sections. Completing the Square Steps To apply the method of completing the square, we will follow a certain set of steps. Given below is the process of completing the square stepwise: Step 1: Write the quadratic equation as x2 + bx + c. (Coefficient of x2 needs to be 1. If not, take it as the common factor.) Step 2: Determine half of the coefficient of x. Step 3: Take the square of the number obtained in step 1. Step 4: Add and subtract the square obtained in step 2 to the x2 term. Step 5: Factorize the polynomial and apply the algebraic identity x2 + 2xy + y2 = (x + y)2 (or) x2 - 2xy + y2 = (x - y)2 to complete the square. How to Apply Completing the Square Method? Now that we have gone through the steps of completing the square in the above section, let us learn how to apply the completing the square method using an example. Example: Complete the square in the expression -4x2 - 8x - 12. Solution: First, we should make sure that the coefficient of x2 is '1'. If the coefficient of x2 is NOT 1, we will place the number outside as a common factor. We will get: -4x2 - 8x - 12 = -4(x2 + 2x + 3) Now, the coefficient of x2 is 1. Step 1: Find half of the coefficient of x. Here, the coefficient of 'x' is 2. Half of 2 is 1. Step 2: Find the square of the above number. 12 = 1 Step 3: Add and subtract the above number after the x term in the expression whose coefficient of x2 is 1. This means, -4(x2 + 2x + 3) = -4(x2 + 2x + 1 - 1 + 3) Step 4: Factorize the perfect square trinomial formed by the first 3 terms using the identity x2 + 2xy + y2 = (x + y)2. In this case, x2 + 2x + 1 = (x + 1)2.The above expression from Step 3 becomes: -4(x2 + 2x + 1 - 1 + 3) = -4((x + 1)2 - 1 + 3) Step 5: Simplify the last two numbers. Here, -1 + 3 = 2. Thus, the above expression is: -4x2 - 8x - 12 = -4(x + 1)2 - 8. This is of the form a(x + m)2 + n. Hence, we have completed the square. Thus, -4x2 - 8x - 12 = -4(x + 1)2 - 8 Note: To complete the square in an expression ax2 + bx + c Make sure the coefficient of x2 is 1. Add and subtract (b/2a)2 after the 'x' term and simplify. Completing the Square Formula Completing the square formula is a technique or method to convert a quadratic polynomial or equation into a perfect square with some additional constant. A quadratic expression in variable x: ax2 + bx + c, where a, b and c are any real numbers but a ≠ 0, can be converted into a perfect square with some additional constant by using completing the square formula or technique. Note: Completing the square formula is used to derive the quadratic formula. Completing the square formula is a technique or method that can also be used to find the roots of the given quadratic equations, ax2 + bx + c = 0, where a, b and c are any real numbers but a ≠ 0. Formula for Completing the Square: The formula for completing the square is: ax2 + bx + c ⇒ a(x + m)2 + n, where m = b/2a and n = c - (b2/4a) Instead of using the complex step-wise method for completing the square, we can use the following simple formula to complete the square. To complete the square in the expression ax2 + bx + c, first find the values of m and n using the above formulas and then substitute these values in: ax2 + bx + c = a(x + m)2 + n. These formulas are derived geometrically. Let us study this in detail using illustrations in the following sections. Completing the Square Formula Examples Here are a few examples of the application of completing the square formula. Example 1: Using completing the square formula, find the number that should be added to x2 - 7x in order to make it a perfect square trinomial. Solution: The given expression is x2 - 7x. Method 1: Comparing the given expression with ax2 + bx + c, a = 1; b = -7 Using the formula, the term that should be added to make the given expression a perfect square trinomial is, (b/2a)2 = (-7/2(1))2 = 49/4. Thus, from both methods, the term that should be added to make the given expression a perfect square trinomial is 49/4. Method 2: The coefficient of x is -7. Half of this number is -7/2. Finding the square, (-7/2)2 = 49/4 Example 2: Use completing the square formula to solve: x2 - 4x - 8 = 0. Solution: Method 1: Using formula, ax2 + bx + c = a(x + m)2 + n. Here, a = 1, b = -4, c = -8 ⇒ m = b/2a = (-4)/2(1) = -2 and, n = c - (b2/4a) = -8 - (-4)2/4(1) = -12 ⇒ x2 - 4x - 8 = (x - 2)2 - 12. ⇒ (x - 2)2 = 12 ⇒ (x - 2) = ±√12 ⇒ x - 2 = ± 2√3 ⇒ x = 2 ± 2√3 Method 2: Let’s transpose the constant term to the other side of the equation: x2 - 4x = 8. Take half of the coefficient of the x-term, which is -4, including the sign, which gives -2. Square -2 to get +4, and add this squared value to both sides of the equation: x2 - 4x + 4 = 8 + 4 ⇒ x2 - 4x + 4 = 12 This process creates a quadratic expression that is a perfect square on the left-hand side of the equation. Simply we can replace the quadratic with the squared-binomial form: (x - 2)2 = 12 Now, we've completed the expression to create a perfect-square binomial, let’s solve: (x - 2)2 = 12 ⇒ (x - 2) = ±√12 ⇒ x - 2 = ± 2√3 ⇒ x = 2 ± 2√3 Answer: Using completing the square method, x = 2 ± 2√3. Derivation of Completing the Square Formula Let us complete the square in the expression ax2 + bx + c using the square and rectangle in Geometry. Based on the method studied earlier, the coefficient of x2 must be made '1' by taking 'a' as the common factor. We get, ax2 + bx + c = a[x2 + (b/a)x + (c/a)]. Now, we will consider the first two terms, x2 and (b/a)x. Let us consider a square of side 'x' (whose area is x2). Let us also consider a rectangle of length (b/a) and breadth (x) (whose area is (b/a)x). Now, divide the rectangle into two equal parts. The length of each rectangle will be b/2a. Attach half of this rectangle to the right side of the square and the remaining half to the bottom of the square. To complete a geometric square, there is some shortage which is a square of side b/2a. The square of area [(b/2a)2] should be added to x2 + (b/a)x to complete the square. But, we cannot just add, we need to subtract it as well to retain the expression's value. Thus, to complete the square: x2 + (b/a) x = x2 + (b/a)x + (b/2a)2 - (b/2a)2 x2 + (b/a) x = x2 + (b/a)x + (b/2a)2 - b2/4a2 Multiplying and dividing (b/a)x with 2 gives, x2 + (2·x·b/2a) + (b/2a)2 - b2/4a2 By using the identity, x2 + 2xy + y2 = (x + y)2 The above equation can be written as, x2 + (b/a) x = (x + b/2a)2 - (b2/4a2) By substituting this in (1): ax2 + bx + c = a((x + b/2a)2 - b2/4a2 + c/a) = a(x + b/2a)2 - b2/4a + c = a(x + b/2a)2 + (c - b2/4a) This is of the form a(x + m)2 + n, where, m = b/2a n = c - (b2/4a) Example: We will complete the square in -4x2 - 8x - 12 using this formula. Comparing this with ax2 + bx + c, a = -4; b = -8; c = -12 Find the values of 'm' and 'n' using: m = b/2a = -8/2(-4) = 1 n = c - (b2/4a) = -12 - (-8)2/4(-4) = -8 Substitute these values in: ax2 + bx + c = a(x + m)2 + n We get: - 4x2 - 8x - 12 = -4(x + 1)2 - 8 Note that we have already obtained the same answer by using step-wise method (not by formula) in the previous section "How to Apply Completing the Square Method?". Trick to Learn Completing the Square Method Here are a few tips for completing the square formula. Step 1: Note down the form we wish to obtain after completing the square: a(x + m)2 + n Step 2: After expanding, we get, ax2 + 2amx + am2 + n Step 3: Compare the given expression, say ax2 + bx + c and find m and n as m = b/2a and n = c - (b2/4a). Challenging Questions: Solve by completing the square: x4 - 18x2 + 17 = 0. Hint: Assume x2 = t. Write the following equation of the form (x - h)2 + (y - k)2 = r2 by completing the square: x2 + y2 - 4x - 6y + 8 = 0. ☛ Related Articles: Cuemath is one of the world's leading math learning platforms that offers LIVE 1-to-1 online math classes for grades K-12. Our mission is to transform the way children learn math, to help them excel in school and competitive exams. Our expert tutors conduct 2 or more live classes per week, at a pace that matches the child's learning needs. Example 1: Use completing the square method to solve: x2 - 4x - 5 = 0. Solution: Let us transpose the constant term to the other side of the equation: x2 - 4x = 5 Now, take half of the coefficient of the x-term, which is -4, including the sign, which gives -2. Take the square of -2 to get +4, and add this squared value to both sides of the equation: x2 - 4x + 4 = 5 + 4 ⇒ x2 - 4x + 4 = 9 By using one of the algebraic identities, we can write x2 - 4x + 4 = (x - 2)2. (x - 2)2 = 9 Now that we have completed the expression to create a perfect-square binomial, let us solve: (x - 2)2 = 9 ⇒ x - 2 = ±3 ⇒ x = 2 ± 3 ⇒ x = 5, -1 Answer: x = 5 or -1. Example 2: Complete the square in the quadratic expression 2x2 + 7x + 6. Solution: The given expression is 2x2 + 7x + 6. The first step to complete the square is to make the coefficient of x2 as 1. We will take the coefficient of x2 (which is 2) as a common factor. 2x2 + 7x + 6 = 2(x2 + (7/2)x + 3) → Equation (1) The coefficient of x is 7/2. Half of it is 7/4. Its square is (7/4)2 = 49/16. [This term can also be found using (b/2a)2 = [7/2(2)]2 = 49/16] Add and subtract it after the x term in Equation (1): 2x2 + 7x + 6 = 2(x2 + (7/2)x + 49/4 - 49/4 + 3) Factorize the trinomial made by the first three terms: 2x2 + 7x + 6 = 2(x2 + (7/2)x + (49/16) - (49/16) + 3) = 2[(x + (7/4))2 - (49/16) + 3] = 2((x + (7/4))2 - (1/16)) = 2(x + (7/4))2 - 1/8 The final answer is of the form a(x + m)2 + n and hence perfecting the square has been done. Answer: 2x2 + 7x + 6 = 2(x + (7/4))2 - (1/8) Example 3: Solve by completing the square x2 - 10x + 16 = 0. Solution: The given quadratic equation is: x2 - 10x + 16 = 0 Here, the coefficient of x2 is already 1. The coefficient of x is (-10). The square of half of it is (-5)2 = 25. Adding and subtracting it on the left-hand side of the given equation after the 'x' term: x2 - 10x + 25 - 25 + 16 = 0 ⇒ (x - 5)2 - 25 + 16 = 0 [∵ x2 - 10x + 25 = (x - 5)2] ⇒ (x - 5)2 - 9 = 0 ⇒ (x - 5)2 = 9 ⇒ (x - 5) = ±√9 [Taking square root on both sides] ⇒ x - 5 = 3 OR x - 5 =

−3 = x = 8; x = 2 ∴ x = 8, 2 Answer: x = 8 or 2. View Answer > go to slidego to slidego to slide Have questions on basic mathematical concepts? Become a problem-solving champ using logic, not rules. Learn the why behind math with our certified experts Book a Free Trial Class FAQs on Completing the Square Completing the square is a method in mathematics that is used for converting a quadratic expression of the form ax2 + bx + c to the vertex form a(x + m)2 + n. The most common use of this method is in solving a quadratic equation which can be done by rearranging the expression obtained after completing the square. What is the Easiest Way to Learn to Complete the Square? The easiest way to learn to complete the square method is using the formula, a(x + m)2 + n = a(x + m)2 + n. Here, m and n can be calculated with the help of the following formulas, m = b/2a and n = c - (b2/4a). Where Can I Find Completing the Square Calculator? Click here to get the completing the square calculator with step-by-step explanation. What is the Use of Completing the Square? Completing the square formula is used for the following purposes: What to Add When Completing the Square? If we have the expression ax2 + bx + c, then we need to add and subtract (b/2a)2 which will complete the square in the expression. This will result in [x + (b/2a)]2 - (b/2a)2 + c. How do you Complete the Square With two Variables? Consider an expression in two variables x2 + y2 + 2x + 4y + 7. To complete the square, we take each of the coefficients of x and y, make their value half, and square it. This means, (1/2 × 2)2 = 1 and (1/2 × 4)2 = 4. Let us add and subtract this to the given equation. Then, rearrange the terms to complete the squares. x2 + y2 + 2x + 4y + 7 + (1 - 1) + (4 - 4) = (x2 + 2x + 1) + (y2 + 4y + 4) + 7 - 1 - 4 = (x + 1)2 + (y + 2)2 + 2 When to use Perfecting the Square? We use the perfecting the square method when we want to convert a quadratic expression of the form ax2 + bx + c to the vertex form a(x - h)2 + k. What is Completing the Square Formula? Completing the square formula is the formula required to convert a quadratic polynomial or equation into a perfect square with some additional constant. It is expressed as, ax2 + bx + c ⇒ a(x + m)2 + n, where m and n are real numbers. What is the Use of Completing the Square Formula? Completing the square formula is used when we want to represent a quadratic polynomial or equation into a perfect square with some additional constant and thus used to factorize a quadratic polynomial. Download Article A walkthrough for the entire process of completing the square Download Article Completing the square is a helpful technique that allows you to rearrange a quadratic equation into a neat form that makes it easier to visualize or even solve. It's used to determine the vertex of a parabola and to find the roots of a quadratic equation. If you're just starting out with completing the square, or if the math isn't exactly adding up, follow along with these easy steps to become a quadratic whiz. To complete the square with x 2 + a x + C = y {\displaystyle x^{2}+ax+C=y} , subtract C from both sides, then add (1 2 a) 2 {\displaystyle \left({\frac {1}{2}}a\right)^{2}} to both sides. Now the equation can be factored as (x + 1 2 a) 2 = y − C + 1 2 a {\displaystyle \left(x+{\frac {1}{2}}a\right)^{2}=y-C+{\frac {1}{2}}a} , where the left side is a perfect square. 1 Write down the equation. Let's say you're working with the following equation: x 2 + 10 x + 5 = 0. {\displaystyle x^{2}+10x+5=0.} 2 Subtract the last term from both sides of the equation. To isolate your x terms, move the constant (number without an "x") to the other side of the equation by subtracting it from both sides. x 2 + 10 x = − 5. {\displaystyle x^{2}+10x=-5.} [1] Advertisement 3 Halve the coefficient of the x term and square it. Take the number in front of the x (second) term in the equation, divide it by 2, and then square it.[2] In this example, the second term is 10 x , {\displaystyle 10x,} so 10 2 = 5 , {\displaystyle {\frac {10}{2}}=5,} and 5 2 = 25 {\displaystyle 5^{2}=25} . 4 Add the term to both sides of the equation. By adding this term to the left side of the equation, you create a polynomial that is a perfect square. To keep the equation balanced, add the number to the right side of the equation, as well.[3] x 2 + 10 x + 25 = − 5 + 25. {\displaystyle x^{2}+10x+25=-5+25.} 5 Factor the left side of the equation. Now that the left side is a perfect square, you can rewrite it as (x + the halved second term)2, or (x + 5) 2 {\displaystyle \left(x+5\right)^{2}} for this example. You can check your math by multiplying it out and seeing if it gives you the first three terms from the last step.[4] 6 Take the square root of both sides. To solve your equation, undo the square on the left side by taking the square root of both sides.[5] (x + 5) 2 = ± 20 {\displaystyle {\sqrt {\left(x+5\right)^{2}}}=\pm {\sqrt {20}}} . 7 Solve to find the roots of the equation. When you take the square root of the right side of the equation, the result is both negative and positive, since negative numbers are also positive when they're squared. To solve this equation, break it into two parts: x = − 5 + 20 {\displaystyle x=-5+{\sqrt {20}}} and x = − 5 − 20 {\displaystyle x=-5-{\sqrt {20}}} .[6] Advertisement 1 Write your equation. For this example, try an equation where the x 2 {\displaystyle x^{2}} term has a coefficient, like 3 x 2 + 4 x − 15 = y {\displaystyle 3x^{2}+4x-15=y} . Vertex form is a way of writing a quadratic equation (a x 2 + b x + C) {\displaystyle \left(ax^{2}+bx+C\right)} with the coordinates of the vertex of the parabola formed by that equation. Vertex form is written as f (x) = a (x − h) 2 + k {\displaystyle f(x)=a\left(x-h\right)^{2}+k} where (h , k) {\displaystyle (h,k)} is the vertex of a parabola.[7] 2 Move the constant to the right side of the equation. Isolate the x terms by adding 15 to both sides of the equation.[8] 3 x 2 + 4 x − 15 + 15 = y + 15 {\displaystyle 3x^{2}+4x-15+15=y+15} 3 Factor out the coefficient of the squared term from the first 2 terms. To complete the square, the leading coefficient has to be 1, so factor 3 out of the left side of the equation.[9] 3 (x 2 + 4 3 x) = y + 15 {\displaystyle 3\left(x^{2}+{\frac {4}{3}}x\right)=y+15} . 4 Halve the second term and square it. The second term, also known as the b term, in the equation, is 4 3 {\displaystyle {\frac {4}{3}}} . First, divide it by two: 4 3 * 1 2 = 4 6 = 2 3 {\displaystyle {\frac {4}{3}}*{\frac {1}{2}}={\frac {4}{6}}={\frac {2}{3}}} . Then, square the term: 2 2 3 2 = 4 9 {\displaystyle {\frac {2^{2}}{3^{2}}}={\frac {4}{9}}} .[10] Completing the square refers to finding a constant "C" that you can add to the first two terms to make a perfect square trinomial, which can be factored into the expression (x + a) 2 {\displaystyle \left(x+a\right)^{2}} . The number you get at this step is the constant that completes the square. 5 Add the term to both sides of the equation. Use the new term to make a perfect square trinomial by adding it into the parentheses.[11] Since you factored out a 3, multiply the new term by three before adding it to the right side of the equation with the y term.[12] 3 (x 2 + 4 3 x + 4 9) = y + 15 + 3 (4 9) 3 (x 2 + 4 3 x + 4 9) = y + 15 + 4 3 3 (x 2 + 4 3 x + 4 9) = y + 49 3 {\displaystyle {\begin{aligned}3\left(x^{2}+{\frac {4}{3}}x+{\frac {4}{9}}\right)&=y+15+3\left({\frac {4}{9}}\right)\\3\left(x^{2}+{\frac {4}{3}}x+{\frac {4}{9}}\right)&=y+15+{\frac {4}{3}}\\3\left(x^{2}+{\frac {4}{3}}x+{\frac {4}{9}}\right)&=y+{\frac {49}{3}}\end{aligned}}} 6 Factor the trinomial as a square and isolate "y." Now that you have a perfect square in the parenthesis, factor it out using the halved b term from before, in this case 2 3 {\displaystyle {\frac {2}{3}}} . Write your equation as a factored square, subtract the constant from the y side, and add it to the x side.[13] 3 (x + 2 3) 2 − 49 3 = y + 49 3 − 49 3 {\displaystyle 3\left(x+{\frac {2}{3}}\right)^{2}-{\frac {49}{3}}=y+{\frac {49}{3}}-{\frac {49}{3}}} 3 (x + 2 3) 2 − 49 3 = y {\displaystyle 3\left(x+{\frac {2}{3}}\right)^{2}-{\frac {49}{3}}=y} If you're using the vertex form f (x) = a (x − h) 2 + k {\displaystyle f(x)=a(x-h)^{2}+k} to find the vertex of a parabola (h, k), remember that "h" has to be negative and "k" has to be positive. In this example, the vertex is (− 2 3 , − 49 3) {\displaystyle \left(-{\frac {2}{3}},-{\frac {49}{3}}\right)} .[14] Advertisement 1 Write down the problem. Let's say you're working with the following equation: 3 x 2 + 4 x + 5 = 6 {\displaystyle 3x^{2}+4x+5=6} 2 Move the constant terms to the left side of the equation. The constant terms are any terms that aren't attached to a variable.[15] In this case, you have 5 on the left side and 6 on the right side, so subtract 6 from both sides of the equation. Now you have 3 x 2 + 4 x − 1 = 0. {\displaystyle 3x^{2}+4x-1=0.} [16] 3 Factor out the coefficient of the squared term. In this case, 3 is the coefficient of the x 2 {\displaystyle x^{2}} term. Divide each term by 3, then put the terms into parenthesis with a 3 in front. So, 3 x 2 ÷ 3 = x 2 {\displaystyle 3x^{2}\div 3=x^{2}} , 4 x ÷ 3 = 4 3 x {\displaystyle 4x\div 3={\frac {4}{3}}x} , and 1 ÷ 3 = 1 3 {\displaystyle 1\div 3={\frac {1}{3}}} . Now the equation is: 3 (x 2 + 4 3 x − 1 3) = 0. {\displaystyle 3\left(x^{2}+{\frac {4}{3}}x-{\frac {1}{3}}\right)=0.} [17] 4 Divide by the constant you just factored out. Since you divided each term by 3, it can be removed without impacting the equation. Now you have x 2 + 4 3 x − 1 3 = 0. {\displaystyle x^{2}+{\frac {4}{3}}x-{\frac {1}{3}}=0} [18] 5 Move the constant to the right side of the equation. Isolate the x terms by moving − 1 3 {\displaystyle -{\frac {1}{3}}} to the other side of the equal sign.[19] x 2 + 4 3 x − 1 3 + 1 3 = 0 + 1 3 {\displaystyle x^{2}+{\frac {4}{3}}x-{\frac {1}{3}}+{\frac {1}{3}}=0+{\frac {1}{3}}} 6 Halve the second term and square it. Next, take the second term, 4 3 {\displaystyle {\frac {4}{3}}} , also known as the b term, and halve and square it to complete the square. When you're done, add it to both sides to keep it balanced.[20] x 2 + 4 3 x + 4 9 = 1 3 + 4 9 {\displaystyle x^{2}+{\frac {4}{3}}x+{\frac {4}{9}}={\frac {1}{3}}+{\frac {4}{9}}} 4 3 * 1 2 = 2 3 {\displaystyle {\frac {4}{3}}*{\frac {1}{2}}={\frac {2}{3}}} 2 2 3 2 = 4 9 {\displaystyle {\frac {2^{2}}{3^{2}}}={\frac {4}{9}}} 7 Write the left side of the equation as a perfect square. Since you've already used a formula to find the missing term, all you have to do is put x and half of the second coefficient in parentheses and square them, like so: (x + 2 3) 2 {\displaystyle \left(x+{\frac {2}{3}}\right)^{2}} . The equation should now read: (x + 2 3) 2 = 7 9 {\displaystyle \left(x+{\frac {2}{3}}\right)^{2}={\frac {7}{9}}} .[21] Note that factoring that perfect square will give you the three terms: x 2 + 4 3 x + 4 9 {\displaystyle x^{2}+{\frac {4}{3}}x+{\frac {4}{9}}} . 8 Take the square root of both sides. On the left side of the equation, the square root of (x + 2 3) 2 {\displaystyle \left(x+{\frac {2}{3}}\right)^{2}} is just x + 2 3 {\displaystyle x+{\frac {2}{3}}} . On the right side, the square root of the denominator, 9, is 3, and the square root of 7 is 7 {\displaystyle {\sqrt {7}}} , so the square root is ± 7 3 {\displaystyle \pm {\frac {\sqrt {7}}{3}}} .[22] Remember to write ± {\displaystyle \pm } because a square root can be positive or negative. 9 Isolate the variable. To isolate the variable x, move the constant term 2 3 {\displaystyle {\frac {2}{3}}} over to the right side of the equation. Now you have two possible answers for x: ± 7 3 − 2 3 {\displaystyle \pm {\frac {\sqrt {7}}{3}}-{\frac {2}{3}}} . You can leave it at that or find the actual square root of 7 if you need an answer without the radical sign.[23] Advertisement Add New Question Question Why do you halve the b value and then square it? It makes no sense to me. It does seem strange and arbitrary, but there is a reason for it. The power move is taking the square root of both sides, but you can't simplify the square root of most polynomials. The step you ask about is a setup move to make the power move work. If I have, for example, x^2 + 4x = 5, and take the square root of both sides, nothing happens, it just makes a mess. But if I add 4 to both sides first and take the square root of both sides of x^2 + 4x + 4 = 9, it simplifies to [x+2] = 3 and the quadratic equation is reduced to a linear equation. Question What's the completing the square formula if x > 1? The value of x doesn't matter. The process remains as shown above. Question In Part 1 of 2, how did you get 11/9 in Step 8? Both sides of that equation are being divided by 3 (to get rid of the coefficient of the first term). Dividing the second term (11/3) by 3 gives us 11/9. See more answers Ask a Question Advertisement Thanks Thanks Advertisement Thanks for reading our article! If you'd like to learn more about math, check out our in-depth interview with David Jia. This article was co-authored by David Jia and by wikiHow staff writer, Carmine Shannon. David Jia is an Academic Tutor and the Founder of LA Math Tutoring, a private tutoring company based in Los Angeles, California. With over 10 years of teaching experience, David works with students of all ages and grades in various subjects, as well as college admissions counseling and test preparation for the SAT, ACT, ISEE, and more. After attaining a perfect 800 math score and a 690 English score on the SAT, David was awarded the Dickinson Scholarship from the University of Miami, where he graduated with a Bachelor's degree in Business Administration. Additionally, David has worked as an instructor for online videos for textbook companies such as Larson Texts, Big Ideas Learning, and Big Ideas Math. This article has been viewed 475,711 times. Co-authors: 32 Updated: July 7, 2025 Views: 475,711 Categories: Algebra Print Send fan mail to authors Thanks to all authors for creating a page that has been read 475,711 times. "I tutor math to high school students. Since everyone learns a little differently, I am always looking for different ways to present material. The second method in this article is an example of presenting the idea a little differently!"..." more Share your story Download Article A walkthrough for the entire process of completing the square Download Article Completing the square is a helpful technique that allows you to rearrange a quadratic equation into a neat form that makes it easier to visualize or even solve. It's used to determine the vertex of a parabola and to find the roots of a quadratic equation. If you're just starting out with completing the square, or if the math isn't exactly adding up, follow along with these easy steps to become a quadratic whiz. To complete the square with x 2 + a x + C = y {\displaystyle x^{2}+ax+C=y} , subtract C from both sides, then add (1 2 a) 2 {\displaystyle \left({\frac {1}{2}}a\right)^{2}} to both sides. Now the equation can be factored as (x + 1 2 a) 2 = y − C + 1 2 a {\displaystyle \left(x+{\frac {1}{2}}a\right)^{2}=y-C+{\frac {1}{2}}a} , where the left side is a perfect square. 1 Write down the equation. Let's say you're working with the following equation: x 2 + 10 x + 5 = 0. {\displaystyle x^{2}+10x+5=0.} 2 Subtract the last term from both sides of the equation. To isolate your x terms, move the constant (number without an "x") to the other side of the equation by subtracting it from both sides. x 2 + 10 x = − 5. {\displaystyle x^{2}+10x=-5.} [1] Advertisement 3 Halve the coefficient of the x term and square it. Take the number in front of the x (second) term in the equation, divide it by 2, and then square it.[2] In this example, the second term is 10 x , {\displaystyle 10x,} so 10 2 = 5 , {\displaystyle {\frac {10}{2}}=5,} and 5 2 = 25 {\displaystyle 5^{2}=25} . 4 Add the term to both sides of the equation. By adding this term to the left side of the equation, you create a polynomial that is a perfect square. To keep the equation balanced, add the number to the right side of the equation, as well.[3] x 2 + 10 x + 25 = − 5 + 25. {\displaystyle x^{2}+10x+25=-5+25.} 5 Factor the left side of the equation. Now that the left side is a perfect square, you can rewrite it as (x + the halved second term)2, or (x + 5) 2 {\displaystyle \left(x+5\right)^{2}} for this example. You can check your math by multiplying it out and seeing if it gives you the first three terms from the last step.[4] 6 Take the square root of both sides. To solve your equation, undo the square on the left side by taking the square root of both sides.[5] (x + 5) 2 = ± 20 {\displaystyle {\sqrt {\left(x+5\right)^{2}}}=\pm {\sqrt {20}}} . 7 Solve to find the roots of the equation. When you take the square root of the right side of the equation, the result is both negative and positive, since negative numbers are also positive when they're squared. To solve this equation, break it into two parts: x = − 5 + 20 {\displaystyle x=-5+{\sqrt {20}}} and x = − 5 − 20 {\displaystyle x=-5-{\sqrt {20}}} .[6] Advertisement 1 Write your equation. For this example, try an equation where the x 2 {\displaystyle x^{2}} term has a coefficient, like 3 x 2 + 4 x − 15 = y {\displaystyle 3x^{2}+4x-15=y} . Vertex form is a way of writing a quadratic equation (a x 2 + b x + C) {\displaystyle \left(ax^{2}+bx+C\right)} with the coordinates of the vertex of the parabola formed by that equation. Vertex form is written as f (x) = a (x − h) 2 + k {\displaystyle f(x)=a\left(x-h\right)^{2}+k} where (h , k) {\displaystyle (h,k)} is the vertex of a parabola.[7] 2 Move the constant to the right side of the equation. Isolate the x terms by adding 15 to both sides of the equation.[8] 3 x 2 + 4 x − 15 + 15 = y + 15 {\displaystyle 3x^{2}+4x-15+15=y+15} 3 Factor out the coefficient of the squared term from the first 2 terms. To complete the square, the leading coefficient has to be 1, so factor 3 out of the left side of the equation.[9] 3 (x 2 + 4 3 x) = y + 15 {\displaystyle 3\left(x^{2}+{\frac {4}{3}}x\right)=y+15} . 4 Halve the second term and square it. The second term, also known as the b term, in the equation, is 4 3 {\displaystyle {\frac {4}{3}}} . First, divide it by two: 4 3 * 1 2 = 4 6 = 2 3 {\displaystyle {\frac {4}{3}}*{\frac {1}{2}}={\frac {4}{6}}={\frac {2}{3}}} . Then, square the term: 2 2 3 2 = 4 9 {\displaystyle {\frac {2^{2}}{3^{2}}}={\frac {4}{9}}} .[10] Completing the square refers to finding a constant "C" that you can add to the first two terms to make a perfect square trinomial, which can be factored into the expression (x + a) 2 {\displaystyle \left(x+a\right)^{2}} . The number you get at this step is the constant that completes the square. 5 Add the term to both sides of the equation. Use the new term to make a perfect square trinomial by adding it into the parentheses.[11] Since you factored out a 3, multiply the new term by three before adding it to the right side of the equation with the y term.[12] 3 (x 2 + 4 3 x + 4 9) = y + 15 + 3 (4 9) 3 (x 2 + 4 3 x + 4 9) = y + 15 + 4 3 3 (x 2 + 4 3 x + 4 9) = y + 49 3 {\displaystyle {\begin{aligned}3\left(x^{2}+{\frac {4}{3}}x+{\frac {4}{9}}\right)&=y+15+3\left({\frac {4}{9}}\right)\\3\left(x^{2}+{\frac {4}{3}}x+{\frac {4}{9}}\right)&=y+15+{\frac {4}{3}}\\3\left(x^{2}+{\frac {4}{3}}x+{\frac {4}{9}}\right)&=y+{\frac {49}{3}}\end{aligned}}} 6 Factor the trinomial as a square and isolate "y." Now that you have a perfect square in the parenthesis, factor it out using the halved b term from before, in this case 2 3 {\displaystyle {\frac {2}{3}}} . Write your equation as a factored square, subtract the constant from the y side, and add it to the x side.[13] 3 (x + 2 3) 2 − 49 3 = y + 49 3 − 49 3 {\displaystyle 3\left(x+{\frac {2}{3}}\right)^{2}-{\frac {49}{3}}=y+{\frac {49}{3}}-{\frac {49}{3}}} 3 (x + 2 3) 2 − 49 3 = y {\displaystyle 3\left(x+{\frac {2}{3}}\right)^{2}-{\frac {49}{3}}=y} If you're using the vertex form f (x) = a (x − h) 2 + k {\displaystyle f(x)=a(x-h)^{2}+k} to find the vertex of a parabola (h, k), remember that "h" has to be negative and "k" has to be positive. In this example, the vertex is (− 2 3 , − 49 3) {\displaystyle \left(-{\frac {2}{3}},-{\frac {49}{3}}\right)} .[14] Advertisement 1 Write down the problem. Let's say you're working with the following equation: 3 x 2 + 4 x + 5 = 6 {\displaystyle 3x^{2}+4x+5=6} 2 Move the constant terms to the left side of the equation. The constant terms are any terms that aren't attached to a variable.[15] In this case, you have 5 on the left side and 6 on the right side, so subtract 6 from both sides of the equation. Now you have 3 x 2 + 4 x − 1 = 0. {\displaystyle 3x^{2}+4x-1=0.} [16] 3 Factor out the coefficient of the squared term. In this case, 3 is the coefficient of the x 2 {\displaystyle x^{2}} term. Divide each term by 3, then put the terms into parenthesis with a 3 in front. So, 3 x 2 ÷ 3 = x 2 {\displaystyle 3x^{2}\div 3=x^{2}} , 4 x ÷ 3 = 4 3 x {\displaystyle 4x\div 3={\frac {4}{3}}x} , and 1 ÷ 3 = 1 3 {\displaystyle 1\div 3={\frac {1}{3}}} . Now the equation is: 3 (x 2 + 4 3 x − 1 3) = 0. {\displaystyle 3\left(x^{2}+{\frac {4}{3}}x-{\frac {1}{3}}\right)=0.} [17] 4 Divide by the constant you just factored out. Since you divided each term by 3, it can be removed without impacting the equation. Now you have x 2 + 4 3 x − 1 3 = 0. {\displaystyle x^{2}+{\frac {4}{3}}x-{\frac {1}{3}}=0} [18] 5 Move the constant to the right side of the equation. Isolate the x terms by moving − 1 3 {\displaystyle -{\frac {1}{3}}} to the other side of the equal sign.[19] x 2 + 4 3 x − 1 3 + 1 3 = 0 + 1 3 {\displaystyle x^{2}+{\frac {4}{3}}x-{\frac {1}{3}}+{\frac {1}{3}}=0+{\frac {1}{3}}} 6 Halve the second term and square it. Next, take the second term, 4 3 {\displaystyle {\frac {4}{3}}} , also known as the b term, and halve and square it to complete the square. When you're done, add it to both sides to keep it balanced.[20] x 2 + 4 3 x + 4 9 = 1 3 + 4 9 {\displaystyle x^{2}+{\frac {4}{3}}x+{\frac {4}{9}}={\frac {1}{3}}+{\frac {4}{9}}} 4 3 * 1 2 = 2 3 {\displaystyle {\frac {4}{3}}*{\frac {1}{2}}={\frac {2}{3}}} 2 2 3 2 = 4 9 {\displaystyle {\frac {2^{2}}{3^{2}}}={\frac {4}{9}}} 7 Write the left side of the equation as a perfect square. 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On the right side, the square root of the denominator, 9, is 3, and the square root of 7 is 7 {\displaystyle {\sqrt {7}}} , so the square root is ± 7 3 {\displaystyle \pm {\frac {\sqrt {7}}{3}}} .[22] Remember to write ± {\displaystyle \pm } because a square root can be positive or negative. 9 Isolate the variable. To isolate the variable x, move the constant term 2 3 {\displaystyle {\frac {2}{3}}} over to the right side of the equation. Now you have two possible answers for x: ± 7 3 − 2 3 {\displaystyle \pm {\frac {\sqrt {7}}{3}}-{\frac {2}{3}}} . You can leave it at that or find the actual square root of 7 if you need an answer without the radical sign.[23] Advertisement Add New Question Question Why do you halve the b value and then square it? It makes no sense to me. It does seem strange and arbitrary, but there is a reason for it. The power move is taking the square root of both sides, but you can't simplify the square root of most polynomials. The step you ask about is a setup move to make the power move work. If I have, for example, x^2 + 4x = 5, and take the square root of both sides, nothing happens, it just makes a mess. But if I add 4 to both sides first and take the square root of both sides of x^2 + 4x + 4 = 9, it simplifies to [x+2] = 3 and the quadratic equation is reduced to a linear equation. Question What's the completing the square formula if x > 1? The value of x doesn't matter. The process remains as shown above. Question In Part 1 of 2, how did you get 11/9 in Step 8? Both sides of that equation are being divided by 3 (to get rid of the coefficient of the first term). Dividing the second term (11/3) by 3 gives us 11/9. See more answers Ask a Question Advertisement Thanks Thanks Advertisement Thanks for reading our article! If you'd like to learn more about math, check out our in-depth interview with David Jia. This article was co-authored by David Jia and by wikiHow staff writer, Carmine Shannon. 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Since everyone learns a little differently, I am always looking for different ways to present material. The second method in this article is an example of presenting the idea a little differently!"..." more Share your story Say we have a simple expression like x2 + bx. Having x twice in the same expression can make life hard. What can we do? Well, with a little inspiration from Geometry we can convert it, like this: As you can see x2 + bx can be rearranged nearly into a square ... ... and we can complete the square with (b/2)2 In Algebra it looks like this: x2 + bx + (b/2)2 = (x+b/2)2 "Complete the Square" So, by adding (b/2)2 we can complete the square. The result of (x+b/2)2 has x only once, which is easier to use. Keeping the Balance Now ... we can't just add (b/2)2 without also subtracting it too! Otherwise the whole value changes. So let's see how to do it properly with an example: Start with: ("b" is 6 in this case) Complete the Square: (Add and subtract the new term) Simplify it and we are done. The result: x2 + 6x + 7 = (x+3)2 − 2 And now x only appears once, and our job is done! A Shortcut Approach Here is a method you may like, it is quick when you get used to it. First think about the result we want: (x+d)2 + e Which can be expanded to get: x2 + 2dx + d2 + e Now see if we can turn our example into that form to discover d and e. Now we can "force" an answer: We know that 6x must end up as 2dx, so d must be 3 Next we see that 7 must become d2 + e = 9 + e, so e must be −2 And we get the same result (x+3)2 − 2 as above! Now, let us look at a useful application: solving Quadratic Equations ... Solving General Quadratic Equations by Completing the Square We can complete the square to solve a Quadratic Equation (find where it is equal to zero). But a general Quadratic Equation may have a coefficient of a in front of x2: ax2 + bx + c = 0 But that is easy to deal with that we divide the whole equation by "a" first, then carry on: x2 + (b/a)x + c/a = 0 Steps Now we can solve a Quadratic Equation in 5 steps: Step 1 Divide all terms by a (the coefficient of x2) Step 2 Move the number term (c/a) to the right side of the equation Step 3 Complete the square on the left side of the equation and balance this by adding the same value to the right side of the equation. We now have something that looks like (x + p)2 = q, which can be solved this way: Step 4 Take the square root on both sides of the equation Step 5 Subtract the number that remains on the left side of the equation to find x Examples OK, some examples will help! Step 1 can be skipped in this example since the coefficient of x2 is 1 Step 2 Move the number term to the right side of the equation: x2 + 4x = -1 Step 3 Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation. (b/2)2 = (4/2)2 = 22 = 4 x2 + 4x + 4 = -1 + 4 (x + 2)2 = 3 Step 4 Take the square root on both sides of the equation: x + 2 = ±√3 = ±1.73 (to 2 decimals) Step 5 Subtract 2 from both sides: x = ±1.73 – 2 = -3.73 or -0.27 And here is an interesting and useful thing. At the end of step 3 we had the equation: (x + 2)2 = 3 It gives us the vertex (turning point) of x2 + 4x + 1: (-2, -3) Step 1 Divide all terms by 5 x2 – 0.8x – 0.4 = 0 Step 2 Move the number term to the right side of the equation: x2 – 0.8x = 0.4 Step 3 Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation: (b/2)2 = (0.8/2)2 = 0.42 = 0.16 x2 – 0.8x + 0.16 = 0.4 + 0.16 (x – 0.4)2 = 0.56 Step 4 Take the square root on both sides of the equation: x – 0.4 = ±√0.56 = ±0.748 (to 3 decimals) Step 5 Subtract (-0.4) from both sides (in other words, add 0.4): x = ±0.748 + 0.4 = -0.348 or 1.148 Why complete the square when we can just use the Quadratic Formula to solve a Quadratic Equation? Well, one reason is given above, where the new form not only shows us the vertex, but makes it easier to solve. There are also times when the form ax2 + bx + c may be part of a larger question and rearranging it as a(x+d)2 + e makes the solution easier, because x only appears once. For example "x" may itself be a function (like cos(z)) and rearranging it may open up a path to a better solution. Also Completing the Square is the first step in the Derivation of the Quadratic Formula Just think of it as another tool in your mathematics toolbox. 364, 1205, 365, 2331, 2332, 3213, 3896, 3211, 3212, 1206 How did I get the values of d and e from the top of the page? Start with Divide the equation by a Put c/a on other side Add (b/2a)2 to both sides "Complete the Square" Now bring everything back... ... to the left side ... to the original multiple a of x2 And you will notice that we have: a(x+d)2 + e = 0 Where: d = b/2a and: e = c − b2/4a Just like at the top of the page! Download Article A walkthrough for the entire process of completing the square Download Article Completing the square is a helpful technique that allows you to rearrange a quadratic equation into a neat form that makes it easier to visualize or even solve. It's used to determine the vertex of a parabola and to find the roots of a quadratic equation. If you're just starting out with completing the square, or if the math isn't exactly adding up, follow along with these easy steps to become a quadratic whiz. To complete the square with x 2 + a x + C = y {\displaystyle x^{2}+ax+C=y} , subtract C from both sides, then add (1 2 a) 2 {\displaystyle \left({\frac {1}{2}}a\right)^{2}} to both sides. Now the equation can be factored as (x + 1 2 a) 2 = y − C + 1 2 a {\displaystyle \left(x+{\frac {1}{2}}a\right)^{2}=y-C+{\frac {1}{2}}a} , where the left side is a perfect square. 1 Write down the equation. Let's say you're working with the following equation: x 2 + 10 x + 5 = 0. {\displaystyle x^{2}+10x+5=0.} 2 Subtract the last term from both sides of the equation. To isolate your x terms, move the constant (number without an "x") to the other side of the equation by subtracting it from both sides. x 2 + 10 x = − 5. {\displaystyle x^{2}+10x=-5.} [1] Advertisement 3 Halve the coefficient of the x term and square it. Take the number in front of the x (second) term in the equation, divide it by 2, and then square it.[2] In this example, the second term is 10 x , {\displaystyle 10x,} so 10 2 = 5 , {\displaystyle {\frac {10}{2}}=5,} and 5 2 = 25 {\displaystyle 5^{2}=25} . 4 Add the term to both sides of the equation. By adding this term to the left side of the equation, you create a polynomial that is a perfect square. To keep the equation balanced, add the number to the right side of the equation, as well.[3] x 2 + 10 x + 25 = − 5 + 25. {\displaystyle x^{2}+10x+25=-5+25.} 5 Factor the left side of the equation. Now that the left side is a perfect square, you can rewrite it as (x + the halved second term)2, or (x + 5) 2 {\displaystyle \left(x+5\right)^{2}} for this example. You can check your math by multiplying it out and seeing if it gives you the first three terms from the last step.[4] 6 Take the square root of both sides. To solve your equation, undo the square on the left side by taking the square root of both sides.[5] (x + 5) 2 = ± 20 {\displaystyle {\sqrt {\left(x+5\right)^{2}}}=\pm {\sqrt {20}}} . 7 Solve to find the roots of the equation. When you take the square root of the right side of the equation, the result is both negative and positive, since negative numbers are also positive when they're squared. To solve this equation, break it into two parts: x = − 5 + 20 {\displaystyle x=-5+{\sqrt {20}}} and x = − 5 − 20 {\displaystyle x=-5-{\sqrt {20}}} .[6] Advertisement 1 Write your equation. For this example, try an equation where the x 2 {\displaystyle x^{2}} term has a coefficient, like 3 x 2 + 4 x − 15 = y {\displaystyle 3x^{2}+4x-15=y} . Vertex form is a way of writing a quadratic equation (a x 2 + b x + C) {\displaystyle \left(ax^{2}+bx+C\right)} with the coordinates of the vertex of the parabola formed by that equation. Vertex form is written as f (x) = a (x − h) 2 + k {\displaystyle f(x)=a\left(x-h\right)^{2}+k} where (h , k) {\displaystyle (h,k)} is the vertex of a parabola.[7] 2 Move the constant to the right side of the equation. Isolate the x terms by adding 15 to both sides of the equation.[8] 3 x 2 + 4 x − 15 + 15 = y + 15 {\displaystyle 3x^{2}+4x-15+15=y+15} 3 Factor out the coefficient of the squared term from the first 2 terms. To complete the square, the leading coefficient has to be 1, so factor 3 out of the left side of the equation.[9] 3 (x 2 + 4 3 x) = y + 15 {\displaystyle 3\left(x^{2}+{\frac {4}{3}}x\right)=y+15} . 4 Halve the second term and square it. The second term, also known as the b term, in the equation, is 4 3 {\displaystyle {\frac {4}{3}}} . First, divide it by two: 4 3 * 1 2 = 4 6 = 2 3 {\displaystyle {\frac {4}{3}}*{\frac {1}{2}}={\frac {4}{6}}={\frac {2}{3}}} . Then, square the term: 2 2 3 2 = 4 9 {\displaystyle {\frac {2^{2}}{3^{2}}}={\frac {4}{9}}} .[10] Completing the square refers to finding a constant "C" that you can add to the first two terms to make a perfect square trinomial, which can be factored into the expression (x + a) 2 {\displaystyle \left(x+a\right)^{2}} . The number you get at this step is the constant that completes the square. 5 Add the term to both sides of the equation. Use the new term to make a perfect square trinomial by adding it into the parentheses.[11] Since you factored out a 3, multiply the new term by three before adding it to the right side of the equation with the y term.[12] 3 (x 2 + 4 3 x + 4 9) = y + 15 + 3 (4 9) 3 (x 2 + 4 3 x + 4 9) = y + 15 + 4 3 3 (x 2 + 4 3 x + 4 9) = y + 49 3 {\displaystyle {\begin{aligned}3\left(x^{2}+{\frac {4}{3}}x+{\frac {4}{9}}\right)&=y+15+3\left({\frac {4}{9}}\right)\\3\left(x^{2}+{\frac {4}{3}}x+{\frac {4}{9}}\right)&=y+15+{\frac {4}{3}}\\3\left(x^{2}+{\frac {4}{3}}x+{\frac {4}{9}}\right)&=y+{\frac {49}{3}}\end{aligned}}} 6 Factor the trinomial as a square and isolate "y." Now that you have a perfect square in the parenthesis, factor it out using the halved b term from before, in this case 2 3 {\displaystyle {\frac {2}{3}}} . Write your equation as a factored square, subtract the constant from the y side, and add it to the x side.[13] 3 (x + 2 3) 2 − 49 3 = y + 49 3 − 49 3 {\displaystyle 3\left(x+{\frac {2}{3}}\right)^{2}-{\frac {49}{3}}=y+{\frac {49}{3}}-{\frac {49}{3}}} 3 (x + 2 3) 2 − 49 3 = y {\displaystyle 3\left(x+{\frac {2}{3}}\right)^{2}-{\frac {49}{3}}=y} If you're using the vertex form f (x) = a (x − h) 2 + k {\displaystyle f(x)=a(x-h)^{2}+k} to find the vertex of a parabola (h, k), remember that "h" has to be negative and "k" has to be positive. In this example, the vertex is (− 2 3 , − 49 3) {\displaystyle \left(-{\frac {2}{3}},-{\frac {49}{3}}\right)} .[14] Advertisement 1 Write down the problem. Let's say you're working with the following equation: 3 x 2 + 4 x + 5 = 6 {\displaystyle 3x^{2}+4x+5=6} 2 Move the constant terms to the left side of the equation. The constant terms are any terms that aren't attached to a variable.[15] In this case, you have 5 on the left side and 6 on the right side, so subtract 6 from both sides of the equation. Now you have 3 x 2 + 4 x − 1 = 0. {\displaystyle 3x^{2}+4x-1=0.} [16] 3 Factor out the coefficient of the squared term. In this case, 3 is the coefficient of the x 2 {\displaystyle x^{2}} term. Divide each term by 3, then put the terms into parenthesis with a 3 in front. So, 3 x 2 ÷ 3 = x 2 {\displaystyle 3x^{2}\div 3=x^{2}} , 4 x ÷ 3 = 4 3 x {\displaystyle 4x\div 3={\frac {4}{3}}x} , and 1 ÷ 3 = 1 3 {\displaystyle 1\div 3={\frac {1}{3}}} . Now the equation is: 3 (x 2 + 4 3 x − 1 3) = 0. {\displaystyle 3\left(x^{2}+{\frac {4}{3}}x-{\frac {1}{3}}\right)=0.} [17] 4 Divide by the constant you just factored out. Since you divided each term by 3, it can be removed without impacting the equation. Now you have x 2 + 4 3 x − 1 3 = 0. {\displaystyle x^{2}+{\frac {4}{3}}x-{\frac {1}{3}}=0} [18] 5 Move the constant to the right side of the equation. Isolate the x terms by moving − 1 3 {\displaystyle -{\frac {1}{3}}} to the other side of the equal sign.[19] x 2 + 4 3 x − 1 3 + 1 3 = 0 + 1 3 {\displaystyle x^{2}+{\frac {4}{3}}x-{\frac {1}{3}}+{\frac {1}{3}}=0+{\frac {1}{3}}} 6 Halve the second term and square it. Next, take the second term, 4 3 {\displaystyle {\frac {4}{3}}} , also known as the b term, and halve and square it to complete the square. When you're done, add it to both sides to keep it balanced.[20] x 2 + 4 3 x + 4 9 = 1 3 + 4 9 {\displaystyle x^{2}+{\frac {4}{3}}x+{\frac {4}{9}}={\frac {1}{3}}+{\frac {4}{9}}} 4 3 * 1 2 = 2 3 {\displaystyle {\frac {4}{3}}*{\frac {1}{2}}={\frac {2}{3}}} 2 2 3 2 = 4 9 {\displaystyle {\frac {2^{2}}{3^{2}}}={\frac {4}{9}}} 7 Write the left side of the equation as a perfect square. Since you've already used a formula to find the missing term, all you have to do is put x and half of the second coefficient in parentheses and square them, like so: (x + 2 3) 2 {\displaystyle \left(x+{\frac {2}{3}}\right)^{2}} . The equation should now read: (x + 2 3) 2 = 7 9 {\displaystyle \left(x+{\frac {2}{3}}\right)^{2}={\frac {7}{9}}} .[21] Note that factoring that perfect square will give you the three terms: x 2 + 4 3 x + 4 9 {\displaystyle x^{2}+{\frac {4}{3}}x+{\frac {4}{9}}} . 8 Take the square root of both sides. On the left side of the equation, the square root of (x + 2 3) 2 {\displaystyle \left(x+{\frac {2}{3}}\right)^{2}} is just x + 2 3 {\displaystyle x+{\frac {2}{3}}} . On the right side, the square root of the denominator, 9, is 3, and the square root of 7 is 7 {\displaystyle {\sqrt {7}}} , so the square root is ± 7 3 {\displaystyle \pm {\frac {\sqrt {7}}{3}}} .[22] Remember to write ± {\displaystyle \pm } because a square root can be positive or negative. 9 Isolate the variable. To isolate the variable x, move the constant term 2 3 {\displaystyle {\frac {2}{3}}} over to the right side of the equation. Now you have two possible answers for x: ± 7 3 − 2 3 {\displaystyle \pm {\frac {\sqrt {7}}{3}}-{\frac {2}{3}}} . You can leave it at that or find the actual square root of 7 if you need an answer without the radical sign.[23] Advertisement Add New Question Question Why do you halve the b value and then square it? It makes no sense to me. It does seem strange and arbitrary, but there is a reason for it. The power move is taking the square root of both sides, but you can't simplify the square root of most polynomials. The step you ask about is a setup move to make the power move work. If I have, for example, x^2 + 4x = 5, and take the square root of both sides, nothing happens, it just makes a mess. But if I add 4 to both sides first and take the square root of both sides of x^2 + 4x + 4 = 9, it simplifies to [x+2] = 3 and the quadratic equation is reduced to a linear equation. Question What's the completing the square formula if x > 1? The value of x doesn't matter. The process remains as shown above. Question In Part 1 of 2, how did you get 11/9 in Step 8? Both sides of that equation are being divided by 3 (to get rid of the coefficient of the first term). Dividing the second term (11/3) by 3 gives us 11/9. See more answers Ask a Question Advertisement Thanks Thanks Advertisement Thanks for reading our article! If you'd like to learn more about math, check out our in-depth interview with David Jia. This article was co-authored by David Jia and by wikiHow staff writer, Carmine Shannon. David Jia is an Academic Tutor and the Founder of LA Math Tutoring, a private tutoring company based in Los Angeles, California. With over 10 years of teaching experience, David works with students of all ages and grades in various subjects, as well as college admissions counseling and test preparation for the SAT, ACT, ISEE, and more. After attaining a perfect 800 math score and a 690 English score on the SAT, David was awarded the Dickinson Scholarship from the University of Miami, where he graduated with a Bachelor's degree in Business Administration. Additionally, David has worked as an instructor for online videos for textbook companies such as Larson Texts, Big Ideas Learning, and Big Ideas Math. This article has been viewed 475,711 times. Co-authors: 32 Updated: July 7, 2025 Views: 475,711 Categories: Algebra Print Send fan mail to authors Thanks to all authors for creating a page that has been read 475,711 times. "I tutor math to high school students. Since everyone learns a little differently, I am always looking for different ways to present material. The second method in this article is an example of presenting the idea a little differently!"..." more Share your story To complete the square for x2+bx+c: Divide b by 2 and write this inside brackets squared as (x + b/2)2 Square the value of b/2 and subtract it. Add the constant term c to this result. The final result is given by (x + b/2)2 + c - (b/2)2. For example, complete the square for y = x2 + 4x + 1. Step 1. Divide the coefficient of x by 2 The coefficient of x is 4. 4 ÷ 2 = 2 and so, we write (x + 2)2. This step is shown with a purple arrow. Step 2. Square the value found in step 1 and subtract it The value found in step 1 is half of the x coefficient. Half of 4 is 2. We square 2 and subtract it. 22 = 4 and so we subtract 4. This step is shown with the green arrow. Step 3. Bring down the constant term. For y = x2 + 4x + 1, the constant term is 1. We add this 1 on to the end. This is shown with the red arrow. Finally, we simplify the constant terms. -4 + 1 = -3 and so we write this as -3. y = x2 + 4x + 1 is written as y = (x + 2)2 - 3. The formula for completing the square for a quadratic, y = x2 + bx + c, is y = (x + b/2)2 + c – (b/2)2. For example, with y = x2 + 6x + 4, b = 6 and c = 4. Therefore b/2 = 3. Therefore (b/2)2 = 9. Substituting these values into the formula, (b/2)2 to 4 - (3)2 to 4 - 9 which equals -5. The final answer is y = (x + 3)2 – 5. Here is an example of completing the square when the value of b is odd. Complete the square for y = x2 + 3x + 3. When the coefficient of x is odd, write it as a fraction over 2 when completing the square. Step 1. Divide the coefficient of x by 2 The coefficient of x is 3. To divide 3 by 2, write this as 3/2. We write this inside the brackets as (x + 3/2)2. This step is written in purple below. Step 2. Square this value and subtract it To square 3/2, square the numerator and denominator separately. 32 = 9 and 22 = 4. We subtract 9/4 from the brackets written in step 1. This step is written in green below. Step 3. Bring down the constant term The constant term of y = x2 + 3x + 3 is the number on its own. It is 3. We add 3 to the equation built so far. This step is shown in red below. Finally, we simplify by collecting the constant terms of -9/4 and + 3. 3 can be written as 12/4. -9/4 + 12/4 = 3/4. Therefore the result is y = (x + 3/2)2 + 3/4. Completing the square is an algebraic method used to rearrange a quadratic equation from y = ax2+bx+c to the form of y = a(x+h)2+k. Completing the square allows us to solve quadratic equations that cannot be factorised and to find the turning point of a quadratic. It is possible to complete the square for any quadratic equation. For example, completing the square with y = x2 + 4x + 1 rearranges the equation to y = (x + 2)2 – 3. It is important to remember that (x + 2)2 – 3 are the same expression, just rearranged. The completing the square formula can be proven algebraically by expanding (x + b/2)2 to get x2 + bx + (b/2)2. This means that any quadratic of the form x2 + bx can be rearranged to (x + b/2)2 – (b/2)2. We can see that . Therefore . Therefore . Adding a constant term of c to both sides of the equation, any quadratic of the form x2 + bx + c can be written as . Completing the square can be shown visually using the following shapes. Consider a square of area x2 and a rectangle of area bx.Divide the rectangle bx in half to form two rectangles of area bx/2. Place these two rectangles around the square of x2. The square of x2 plus the two rectangles almost form a square with side lengths of x + b/2. However, there is a small square of side length b/2 missing. The area of this small square would be (b/2)2. The area of the overall square (outlined in red above) would be (x + b/2)2. The area of the purple square above would be (b/2)2. Therefore we can say that the overall area is found by (x + b/2)2 – (b/2)2 and this is equal to x2 + bx. Adding a constant term of c to each side of the equation tells us that . To complete the square with a coefficient of x2: Factorise the coefficient Complete the square inside the bracket Simplify the terms inside the bracket Expand the brackets For example, complete the square for y = 2x2 + 8x – 6. Step 1. Factorise the coefficient of x2 is 2. We factorise the expression by bringing a 2 in front of the brackets and dividing every term inside the brackets by 2. We get y = 2[x2 + 4x – 3] Step 2. Complete the square inside the bracket Inside the bracket of y = 2[x2 + 4x - 3] is x2 + 4x – 3. We complete the square for this. i) We halve the x coefficient. Half of 4 is 2 so we get (x + 2)2 ii) We subtract this value squared. 22 is 4 so we get (x + 2)2 – 4. iii) We bring the constant term down. The constant term is -3 so we get (x + 2)2 – 4 – 3. Step 3. Simplify the terms inside the bracket We can simplify (x + 2)2 – 4 – 3 to (x + 2)2 - 7. Step 4. Expand the bracket We have y = 2[(x + 2)2 – 7]. To expand this, we multiply the (x + 2)2 and the -7 term both by 2. We get y = 2(x + 2)2 – 14. In this example we will complete the square with a negative coefficient of x2. Complete the square for y = -3x2 + 6x – 9. In this example, the coefficient of x2 is -3. Factorise the coefficient of x2 We factorise the -3 from each term by writing -3 in front of the brackets and dividing each term within the brackets by -3. Because we are factorising a negative number, the signs of the terms inside the bracket will switch from + to - or - to +, respectively. We get y = -3[x2 – 2x + 3]. Step 2. Complete the square inside the bracket Inside the bracket of y = -3[x2 – 2x + 3] is x2 – 2x + 3. We complete the square for this. i) We halve the x coefficient. The coefficient of x is -2 so we get (x – 1)2 ii) We subtract this value squared. (-1) squared is 1 so we get (x – 1)2 – 1. iii) We bring down the constant term. The constant term is 3 so we get (x – 1)2 – 1 + 3. Step 3. Simplify terms inside the bracket Inside the bracket, (x – 1)2 – 1 + 3 can be simplified to (x – 1)2 + 2. Step 4. Expand the bracket We have y = -3[(x – 1)2 + 2]. To expand this, we multiply the (x – 1)2 term and the +2 term both by -3. We get y = -3(x – 1)2 – 6. Any quadratic equation can be solved by completing the square. After completing the square, rearrange the equation so that there are only terms on one side of the equation and the squared term is equal to (x + a)2 = k. Then square root both sides to get x + a = ±√k. Finally, subtracting a from both sides gives the solution as x = -a ±√k. For example, solve the equation x2 + 8x + 5 = 0. Step 1. Complete the square i) Halve the coefficient of x. The coefficient of x is 8 and half of 8 is 4. We start with writing (x + 4)2. ii) Square this value and subtract it. 4 squared is 16, so we subtract 16 to get (x + 4)2 – 16 iii) Bring down the constant term. The constant term is 5, so we get (x + 4)2 – 16 + 5. This simplifies to (x + 4)2 – 11. Step 2. Solve for x i) First separate the squared bracket and any constant terms on either side of the equals sign. We add 11 to both sides to get (x + 4)2 = 11. ii) Square root both sides of the equation. The square root of (x + 4)2 is just x + 4. The square root of 11 is ±√11. We need to remember to take both the positive and negative solutions. iii) Solve for x We have x + 4 = ±√11. We subtract 4 from both sides to get x = -4 ±√11. Here is another example of solving a quadratic equation by completing the square. For example, solve the equation x2 + 6x – 1 = 0. Step 1. Complete the square i) Halve the coefficient of x. The coefficient of x is 6 and half of 6 is 3. We start with writing (x + 3)2. ii) Square this value and subtract it. 3 squared is 9, so we subtract 9 to get (x + 3)2 – 9 iii) Bring down the constant term. The constant term of x2 + 6x – 1 is -1, so we subtract 1 to get (x + 3)2 – 10. Step 2. Solve for x i) First separate the squared bracket and any constant terms on either side of the equals sign. We add 10 to both sides to get (x + 3)2 = 10. ii) Square root both sides of the equation. The square root of (x + 3)2 is just x + 3. The square root of 10 is ±√10. We need to remember to take both the positive and negative solutions. iii) Solve for x We have x + 3 = ±√10. We subtract 3 from both sides to get x = -3 ±√10. When you complete the square, the quadratic equation is rewritten in the form of y = a(x + h)2 + k. The turning point of a parabola in this form is found at (-h, k). For example, the parabola y = 2(x + 3)2 + 1 has a vertex at the coordinate (-3, 1). The vertex is the turning point of a parabola. The graph below shows the vertex of y = 2(x + 3)2 + 1 at (-3, 1). The vertex is found at the x value that causes the brackets to be equal to zero. In the equation y = 2(x + 3)2 + 1, the brackets are squared. When we square a value, the result is always positive. Therefore the smallest value that can be output by the equation y = 2(x + 3)2 + 1 is when the bracket is worth zero. The value of x that causes this to happen is at x = -3. When x = -3, the equation of y = 2(x + 3)2 + 1 becomes y = 1. Therefore we find the minimum value at x = -3, y = 1 or written as a coordinate (-3, 1). Here is an example of finding the vertex of a quadratic by completing the square. For example, find the vertex of y = x2 – 2x + 3 Step 1. Complete the square i) Halve the coefficient of x. The coefficient of x is -2 and half of -2 is -1. We start with writing (x – 1)2. ii) Square this value and subtract it (-1) squared is 1, so we subtract 1 to get (x – 1)2 – 1 iii) Bring down the constant term. The constant term of y = x2 – 2x + 3 is 3, so we add 3 to get (x – 1)2 + 2. Step 2. Read the vertex from the complete the square form The x coordinate of the vertex is the value of x that makes the bracket equal to zero. (x – 1) is equal to zero when x = 1. Therefore the x coordinate of the vertex is at x = 1. The y coordinate of the vertex is simply the constant term in y = (x – 1)2 + 2. This is just the +2 at the end. The y coordinate of the vertex is at y = 2. Therefore, the vertex is at (1 , 2). Notice how the value of the x coordinate is opposite in sign to the sign written in brackets but the y coordinate is the same sign as the constant term at the end. Here are some examples of reading the turning point from an equation in complete the square form. EquationCoordinates of the Vertex y = a(x + h)2 + k (-h, k) y = (x + 3)2 + 5 (-3, 5) y = 5(x + 1)2 + 2 (-1, 2) y = (x – 3)2 + 7 (3, 7) y = 2(x – 4)2 – 3 (4, -3) y = (x + 1)2 – 6 (-1, -6) (2, -6) y = – 5(x – 5)2 + 5 (5, 0) "Completing the Square" is where we take ... For those of you in a hurry, I can tell you that: But if you have time, let me show you how to "Complete the Square" yourself. Say we have a simple expression like x2 + bx. Having x twice in the same expression can make life hard. What can we do? Well, with a little inspiration from Geometry we can convert it, like this: As you can see x2 + bx can be rearranged nearly into a square ... ... and we can complete the square with (b/2)2 In Algebra it looks like this: x2 + bx + (b/2)2 = (x+b/2)2 "Complete the Square" So, by adding (b/2)2 we can complete the square. The result of (x+b/2)2 has x only once, which is easier to use. Keeping the Balance Now ... we can't just add (b/2)2 without also subtracting it too! Otherwise the whole value changes. So let's see how to do it properly with an example: Start with: ("b" is 6 in this case) Complete the Square: (Add and subtract the new term) Simplify it and we are done. The result: x2 + 6x + 7 = (x+3)2 − 2 And now x only appears once, and our job is done! A Shortcut Approach Here is a method you may like, it is quick when you get used to it. First think about the result we want: (x+d)2 + e Which can be expanded to get: x2 + 2dx + d2 + e Now see if we can turn our example into that form to discover d and e. Now we can "force" an answer: We know that 6x must end up as 2dx, so d must be 3 Next we see that 7 must become d2 + e = 9 + e, so e must be −2 And we get the same result (x+3)2 − 2 as above! Now, let us look at a useful application: solving Quadratic Equations ... Solving General Quadratic Equations by Completing the Square We can complete the square to solve a Quadratic Equation (find where it is equal to zero). But a general Quadratic Equation may have a coefficient of a in front of x2: ax2 + bx + c = 0 But that is easy to deal with that we divide the whole equation by "a" first, then carry on: x2 + (b/a)x + c/a = 0 Steps Now we can solve a Quadratic Equation in 5 steps: Step 1 Divide all terms by a (the coefficient of x2) Step 2 Move the number term (c/a) to the right side of the equation Step 3 Complete the square on the left side of the equation and balance this by adding the same value to the right side of the equation. We now have something that looks like (x + p)2 = q, which can be solved this way: Step 4 Take the square root on both sides of the equation Step 5 Subtract the number that remains on the left side of the equation to find x Examples OK, some examples will help! Step 1 can be skipped in this example since the coefficient of x2 is 1 Step 2 Move the number term to the right side of the equation: x2 + 4x = -1 Step 3 Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation. (b/2)2 = (4/2)2 = 22 = 4 x2 + 4x + 4 = -1 + 4 (x + 2)2 = 3 Step 4 Take the square root on both sides of the equation: x + 2 = ±√3 = ±1.73 (to 2 decimals) Step 5 Subtract 2 from both sides: x = ±1.73 – 2 = -3.73 or -0.27 And here is an interesting and useful thing. At the end of step 3 we had the equation: (x + 2)2 = 3 It gives us the vertex (turning point) of x2 + 4x + 1: (-2, -3) Step 1 Divide all terms by 5 x2 – 0.8x – 0.4 = 0 Step 2 Move the number term to the right side of the equation: x2 – 0.8x = 0.4 Step 3 Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation: (b/2)2 = (0.8/2)2 = 0.42 = 0.16 x2 – 0.8x + 0.16 = 0.4 + 0.16 (x – 0.4)2 = 0.56 Step 4 Take the square root on both sides of the equation: x – 0.4 = ±√0.56 = ±0.748 (to 3 decimals) Step 5 Subtract (-0.4) from both sides (in other words, add 0.4): x = ±0.748 + 0.4 = -0.348 or 1.148 Why complete the square when we can just use the Quadratic Formula to solve a Quadratic Equation? Well, one reason is given above, where the new form not only shows us the vertex, but makes it easier to solve. There are also times when the form ax2 + bx + c may be part of a larger question and rearranging it as a(x+d)2 + e makes the solution easier, because x only appears once. For example "x" may itself be a function (like cos(z)) and rearranging it may open up a path to a better solution. Also Completing the Square is the first step in the Derivation of the Quadratic Formula Just think of it as another tool in your mathematics toolbox. 364, 1205, 365, 2331, 2332, 3213, 3896, 3211, 3212, 1206 How did I get the values of d and e from the top of the page? Start with Divide the equation by a Put c/a on other side Add (b/2a)2 to both sides "Complete the Square" Now bring everything back... ... to the left side ... to the original multiple a of x2 And you will notice that we have: a(x+d)2 + e = 0 Where: d = b/2a and: e = c − b2/4a Just like at the top of the page!

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