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Post Views: 1,764In this tutorial, we will learn how to write the permutation and combination in LaTeX.I will show you both of those one by one.Permutation in LaTeXWe will start with writing the permutation formula in LaTeX.\documentclass{article}\usepackage{amsmath}\begin{document}The number of permutations of n objects taken r at a time is given by:\[_{n}P_{r} = \frac{{n!}}{{(n-r)!}}\]\end{document}Output:The most popular form of permutation can be written in LaTeX like this:\documentclass{article}\usepackage{amsmath}\begin{document}Permutation formula:\[^nC_r = \frac{{n!}}{{(n-r)!}}\]\end{document}Output:You can see the space between n and C is a little bit higher. We can reduce this by using \!just like this:^n\!C\!_r = \frac{{n!}}{{(n-r)!}}We can represent the permutation in many different ways. Lets check one by one:\documentclass{article}\usepackage{amsmath}\begin{document}Permutation Formula:\[P(n,r) = \frac{{n!}}{{(n-r)!}}\]\end{document}Output:We can also represent permutation like this:\documentclass{article}\usepackage{amsmath}\begin{document}Permutation Formula:\[P^n_r = \frac{{n!}}{{(n-r)!}}\]\end{document}Output:Here is the last one.P_n,_r = \frac{{n!}}{{(n-r)!}}Output:Combination in LaTeX\documentclass{article}\usepackage{amsmath}\begin{document}Combination formula:\[^nC_k = \frac{{n!}}{k!{(n-k)!}}\]\end{document}Other forms of combination:\documentclass{article}\usepackage{amsmath}\begin{document}Combination formula:\[\binom{n}{k}={^n}C_k = \frac{{n!}}{k!{(n-k)!}}\]\end{document}Output:You can check:How to write binomial coefficient in LaTeX? This error can be triggered in different ways and, especially for those new to LaTeX, its not always immediately obvious why it happened, or, more importantly, how to resolve it. Well start with an overview of the general conditions which trigger this error then list some of the more common causes and how to fix them. The latter parts of this article offer more in-depth material for readers wishing to broaden their understanding.Note: In this article we use the terms TeX, LaTeX and TeX engine but if you arent sure of their different meanings you can find out in the Overleaf article What's in a Name: A Guide to the Many Flavours of TeX.The main causes of Missing $ insertedThere are three main reasons why (La)TeX generates this error:Youve made an explicit error in your math markup, such as writing $y=f(x)$$we look at other examples later in this article.TeX has detected a character or command designed to be used only when TeX is typesetting mathematics but that character or command has been used when TeX was not typesetting mathematics.TeXnically speaking: TeX detected a character or command designed to work inside math mode but youve tried to use it outside math mode. To clarify the issue with characters designed to work in math mode, what we really mean is using characters assigned certain category codes designed to operate inside mathematical material.TeX has detected a command designed to be used only when TeX is not typesetting mathematics but that command was detected (used) whilst TeX was typesetting mathematical material.TeXnically speaking: TeX detected a command designed to work outside math mode but youve tried to use it inside math mode.Examples of errors and their solutionsThe following examples demonstrate some ways in which the Missing $ inserted error can be triggered, together with providing solutions to resolve the error. Symbol commands must be used in math modeMany math symbols in LaTeX are accessed using commands which must only be used when TeX is typesetting math; i.e., at a time when TeX is in math mode. Examples of commonly-used symbol commands include those for Greek letters: \alpha \((\alpha)\), \beta \((\beta)\), \gamma (\(\gamma)\), \delta \((\delta)\), \Delta \((\Delta)\) and so forth. Many other LaTeX commands, such as those for modifiers: \vec{x} \((\vec{x})\), \tilde{x} \((\tilde{x})\), \hat{x} \((\hat{x})\) etc., are also designed for use in math mode.Using modifiers, symbol commandsand many other math-related commandsoutside of typesetting mathematical content will generate a Missing $ inserted error and force the compiler to enter into math mode.The following example shows what happens if you try to use the command \alpha \((\alpha)\) outside of math (mode):Writing \verb|\alpha| outside math mode will generate an error: \alpha and so this text will not be typeset correctly... Open this error-producing example in OverleafThe following image shows part of the output produced by the LaTeX code above, demonstrating the error caused by using \alpha outside of math mode:Fixing errors caused by LaTeX symbol commandsTo use modifiers, or Greek math symbols, within sentences they must always be wrapped in single dollar signs $...$, or LaTeXs \(...\) syntax, in order for TeX to process them in inline math mode, as shown below.When writing the Greek letter alpha in a sentence, it must be written as \verb|α| to generate α, or as \verb|\(\alpha\)| which also generates \(\alpha\). When writing a vector x in a sentence, it must be written as \verb|$\vec{x}$| to produce $\vec{x}$ or as \verb|\(\vec{x}\)| which also yields \(\vec{x}\). Open this example in Overleaf A list of LaTeX symbolsThe Overleaf article List of Greek letters and math symbols provides a list of symbols which are exclusive to math mode, together with links to further useful resources.Using math-mode-only characters outside math modeTraditionally, TeX/LaTeX reserve certain common characters for use within math mode:^: reserved for creating superscripts_: reserved for creating subscripts$: reserved for starting/stopping math modeUsing them directly outside math mode triggers errors. To type $ outside math mode use \$To type _ outside math mode use _Multiple ways to type ^ outside math mode. A list is provided on tex.stackexchangeUsing underscores outside of math modeA common cause of the Missing $ inserted error is using underscores (_), a math mode character, outside of math modesuch as underscores present in filenames: an example of this is shown below.Using a math character, such as an underscore, in a file name: math_example.tex. Open this error-generating code on OverleafBecause the underscore character is assigned category code 8 it is reserved for creating subscripts when the TeX engine is in math mode. Consequently, when (La)TeX detects the _ in the file name math_example.tex it does so outside of math mode, which triggers an error and results in erroneous typesetting of subsequent text:\(\text{Using a math character, such as an underscore, in a file name: math}_example.tex.\)The _character (technically its category code of 8) triggers an error and causes TeX to enter math mode: the letter e following immediately after the _ is treated as a character to be typeset as a subscript. Processing then continues in math mode resulting in the italicized text and, additionally, there is no way here for TeX to gracefully exit math mode, which will trigger further errors.In order to avoid this particular error, you must always use _ in math modei.e., inside $..$, $$...$$ or, preferably, inside LaTeXs notation of \(...\) and \[...\]. Outside math mode you need to write _ to use or typeset an underscore, as shown in the corrected version of the example: To use the math-mode underscore character in a file name, write it like this: math_example.tex. Open this corrected version on Overleaf Using underscores in URLsYou may also encounter this error when trying to typeset URLs with underscores, e.g. . Instead of escaping each underscore character, you may want to load the url or hyperref package, and then use the \url command like this:\documentclass{article}\usepackage{url} \begin{document}\url{ \end{document} Open this example in Overleaf Underscores in .bib file URLsIf such URLs are in your bibliography .bib file, causing errors to be reported from the .bbl file, then make sure you use the url or doi fields to record these fields in the .bib file:doi = {10.1007/978-94-015-6859-3_4},url = { and load the url or hyperref package in your preamble, if necessary. Most bibliography style files will then be able to automatically wrap these values in a \url{...} command.Using commands not permitted in math modeA number of low-level built-in TeX commands (called primitives) are not permitted in math mode and their use in math mode will trigger the Missing $ inserted error. Although most users are unlikely to use these commands in day-to-day LaTeX code, we make a note here because its possible they could be contained within LaTeX commands (macros) being used in math mode.The following example tries to use the TeX command \vskip inside mathematical material. This is not permitted and generates an error.\documentclass{article}\begin{document}I want to add some space, but this is not the way to do it...\[y=f(x) \vskip5pt z=f(y)\]$y=f(x) \vskip5pt z=f(y)$\end{document} Open this error-generating code on OverleafOther TeX (primitive) commands not permitted when TeX is in math mode include \par, \hrule, \unvbox, \unvcopy and \valign.Blank lines in mathematics\documentclass{article}\begin{document}\begin{equation}y=x^3, z=x^3\end{equation}\end{document} Open this error-generating code on OverleafCause: The blank line between the formulae is converted to a \par command which is not allowed in math mode. You can also see this by writing\documentclass{article}\begin{document}\[y=x^3,\par z=x^3\]\end{document} Open this error-generating code on OverleafTo fix this, either delete the blank lines or comment them out:\documentclass{article}\begin{document}\begin{equation}y=x^3,% This will suppress the blank linez=x^3\end{equation}\end{document} Open this corrected version on Overleaf Using $ inside math environmentsSome LaTeX environments, such as the align and equation environments, do not require math to be wrapped in $ signs, or use of LaTeXs math syntax: \(...\) or \[...\]. The LaTeX code which implements those environments takes care of entering and exiting math mode.The following example shows use of the $ sign inside an amsmath align environment, which triggers a Missing $ inserted error, among many others.... \documentclass{article}\usepackage{amsmath}\begin{document}\begin{align*}$2x - 5y &= 8 \\ 3x + 9y &= -12$\end{align*}\end{document} Open this error-generating code on OverleafThe correct way to write these equations is:\documentclass{article}\usepackage{amsmath}\begin{document}\begin{align*}2x - 5y &= 8 \\ 3x + 9y &= -12\end{align*}\end{document} Open this corrected version on Overleaf Background to the Missing $ inserted errorThe following section is for readers wishing to better understand the reasons behind the error Missing $ inserted. Its not essential reading but it may assist you with finding and fixing errors.There are no missing $ signs but I still get the errorIn some circumstances the Missing $ inserted error can be very confusing because your LaTeX code might not actually have any problems with visibly missing $ characters. For example, the LaTeX fragment \(\verb|$$y=f(x)\par$$|\) superficially looks correct: the \(\verb|$$|\) pairs are balanced but it will trigger the missing $ error. Here, it is due to the \par command which is not allowed in math mode:\documentclass{article}\begin{document}This example generates the error \verb|Missing $ inserted|: $$y=f(x)\par$$\end{document} Open this to see a Missing $ inserted errorThe above LaTeX code actually triggers a cascade of errors, as shown below, so clearly you should not use \par inside math!! Missing $ inserted. $l.12 $$y=f(x)\par $$Ive inserted a begin-math/end-math symbol since I thinkyou left one out. Proceed, with fingers crossed.! Display math should end with $$. \par l.12 $$y=f(x)\par $$The `$' that I just saw supposedly matches a previous `$$'.So I shall assume that you typed `$$' both times.! Missing $ inserted. $l.13 \end{document} I've inserted a begin-math/end-math symbol since I thinkyou left one out. Proceed, with fingers crossed.! Display math should end with $$. \par l.13 \end{document} The `$' that I just saw supposedly matches a previous `$$'.So I shall assume that you typed `$$' both times.When the Missing $ inserted error occurs the TeX engine software is trying to recover and get back on track so it can continue processing after the point where the error occurred. The section Advanced: An explanation of TeXs error recovery mechanism will help you understand why this cascade of errors arises due to TeXs attempts at trying to fix the initial error.Of modes, and whenInternally, TeX engines operate using three states of mind, called modes, which depend on the type of material a TeX engine is currently typesetting. The mode a TeX engine is in at any point during typesetting is called its current mode, which changes throughout the process of typesetting the content of your LaTeX document. For each of a TeX engines modes there are certain commands and characters (or, more correctly, category codes) which are incompatible with TeXs current mode: they shouldnt be used whilst TeX is in that specific mode. If you try to use those inappropriate characters or commands TeX will issue an error such as Missing $ inserted to tell you something is wrong.Math modeThis error Missing $ inserted is related to TeXs math modei.e., the mode a TeX engine is in when you ask LaTeX to typeset some maths. Just for completeness well note there are two types of math mode, reflecting the creation of inline or display math:inline math modedisplay math modeTeX needs these two different math modes because it applies different rules for spacing, symbols sizes etc when typesetting math destined for inline or display.Triggering TeX math modesThere are multiple ways to trigger a TeX engine to enter and then leave math mode. You can use explicit markup such as:LaTeX syntax: \(...\) to enter \(then leave \) inline math mode or \[...\] to enter \[then leave \] display math mode;(historic) TeX syntax: $...$ to enter (first $) then leave (second $) inline math mode or $$...$$ to enter (first $$ pair) then leave (second $$ pair) display math mode;or any one of the LaTeX math environments:\begin{align}...\end{align} etc. Behind the scenes these environment take care of entering and leaving math mode.The actual text of the message Missing $ inserted is built into (hardcoded in) TeX engine software, which is why you will still see Missing $ even if you are not using $ characters to typeset mathematics and use purely LaTeX syntax to markup the math in your document. This can be confusing to new users of LaTeX but theres little that can be done to change this error message text without modifying the source code of TeX engines!Macros: testing for math modeIts possible to test if TeX is currently in math mode using the primitive command \ifmmode; that way you can write macros whose behavior can adapt to avoid generating mode-related errors. Here is a very basic example to demonstrate the principle, which prints Yes. or No. depending whether or not TeX is in math mode at the point of executing the macro.\documentclass{article}\begin{document}ewcommand{\mytest}{\ifmmode \mathrm{Yes}\else No\fi.} Is the macro being used in math mode? $\mytest$ Is the macro being used in math mode? $$\mytest$$ Is the macro being used in math mode? \(\mytest\) Is the macro being used in math mode? \[\mytest\] Is the macro being used in math mode? \mytest\end{document} Open this example in OverleafThe following graphic shows the output produced by the example above:Advanced: An explanation of TeXs error recovery mechanismThe error message Missing $ inserted is not output by Overleaf, LaTeX, or LaTeX packages; it actually originates from within the executable program responsible for typesetting your LaTeX document: that executable program is called a TeX engineyou can think of it as driving the typesetting process.Within the source code of the TeX software, its author (Donald Knuth), makes this observation about the code responsible for generating the Missing $ inserted error: Here is a list of cases where the user has probably gotten into or out of mathmode by mistake. TeX will insert a dollar sign and rescan the current token.At the heart of the Missing $ inserted error is TeX detecting something that should not have been used inside math mode, or something expressly designed for math typesetting being used outside math mode. The task facing TeX is: How do I recover from this? TeX does exactly what Knuth writes: it inserts a dollar sign and rescans the current tokena token is TeXs internal numeric (integer) value which represents the character or command it has just read in. However, due to the precise context of the error, this strategy may, or may not, be successfulas the error text goes on to say Proceed, with fingers crossed.!Worked exampleLets explore Knuths comments by taking a closer look at the following example: \(\verb|$y=f(x)$$\vskip3pt|\). If you open the code below youll see it triggers the error Missing $ inserted.Writing \verb|$y=f(x)$$\vskip3pt| produces...$y=f(x)$$\vskip3pt Start new line... Open this example in OverleafTo review why this triggers an error lets write subscripts to identify each \(\verb|$|\) in the expression to obtain \(\verb|$|_{\mathtt1}\verb|y=f(x)|\verb|$|_{\mathtt2}\verb|$|_{\mathtt3}\verb|\vskip3pt|\).TeX is able to correctly process the first part \(\verb|$|_{\mathtt1}\verb|y=f(x)|\verb|$|_{\mathtt2}\) which is treated as a correctly formatted piece of inline math, producing \(y=f(x)\). Immediately after processing \(\verb|$|_{\mathtt2}\) TeX temporarily exits inline math modein our example we are creating inline math in a paragraph so it briefly enters so-called horizontal mode. Its what happens next that triggers the error. TeX continues to process the \(\verb|$|_{\mathtt3}\) which triggers TeX to re-enter inline math mode. TeX now reads the next token which is the \(\verb|\vskip|\) commandit has not yet read the \(\verb|3pt|\). At this point, TeX sees \(\verb|\vskip|\) but its in inline math mode: \(\verb|\vskip|\) is not allowed there so it triggers TeXs error-handling process as described by Knuth:TeX will insert a dollar sign and rescan the current token.Here, the current token is the \(\verb|\vskip|\) command so what TeX does is place a new $ into its input, lets call it \(\verb|$|_{\mathtt4}\). At this point, TeX is still in inline math mode but now it goes back to read the equivalent of \(\verb|$|_{\mathtt4}\verb|\vskip|\) which is read in inline math mode. The \(\verb|$|_{\mathtt4}\), inserted by TeX itself, now closes the current inline math mode and TeX goes on to read the \(\verb|\vskip|\) command outside inline math modehere, TeX is in a paragraph so the \(\verb|\vskip3pt|\) causes the current paragraph to be ended and \(\verb|3pt|\) of space placed after it. LaTeX specific issues not fitting into one of the other forums of this category. bracketman Posts: 1 Joined: Fri Jan 05, 2024 8:25 am Post by bracketman Fri Jan 05, 2024 8:39 am Hey guys,Please how is this expression written in markdown; factorial.png (23.27 KiB) Viewed 9262 times Stefan Kottwitz Site Admin Posts: 10315 Joined: Mon Mar 10, 2008 9:44 pm Post by Stefan Kottwitz Sat Jan 06, 2024 9:35 am Hi Bracketman, welcome to the forum!I guess you mean LaTeX and not Markdown? With LaTeX, I would use \binom from amsmath: Code: Select all\documentclass{article}\usepackage{amsmath}\begin{document}\[\binom{n}{k} = {}^{52}C_2 = \cdots\]\end{document} Stefan LaTeX.org admin \documentclass{beamer}\mode{\usetheme{Madrid}%\usecolortheme{beaver}}\usepackage[utf8]{inputenc}\usepackage{default}\usepackage[portuguese]{babel}\usepackage{pgfplots}\pgfplotsset{/pgf/number format/use comma,compat=newest}\usepackage{color}\usepackage{amsmath,amsfonts,amssymb}\usepackage{hyperref}\usepackage{tikz}\usepackage{enumerate}\usebackgroundtemplate{%\tikzode[opacity=0.1] {\includegraphics[height=\paperheight,width=\paperwidth]{tu.jpg}};}\title{COUNTING PRINCIPLES, PERMUTATION AND COMBINATION}\author{ \bf HEMANGA DUTTA}\institute[]{\includegraphics[height=1in]{tu.jpg}{\vspace{.3cm}}\\ \bf M.Sc IN MATHEMATICS{\vspace{.3cm}} \\ ROLL NO. : MSM14039{\vspace{.3cm}}\\ {\bf TEZPUR UNIVERSITY}}\date{\hspace{8cm}}\begin{document}\begin{frame} \maketitle \end{frame}%\begin{frame}%\frametitle{Sumrio} %\tableofcontents%\end{frame}\section{}\begin{frame}\frametitle{OUTLINE} In this presentation, we are going to discuss about, \begin{enumerate}\item {\bf Four Basic Counting Principles} \begin{enumerate} \item Addition \item Multiplication\item Subtraction\item Division{\vspace{.5cm}}\end{enumerate}\item {\bf Permutation} \begin{enumerate}\item Permutation of Multiset{\vspace{.5cm}}\end{enumerate}\item {\bf Combination} \begin{enumerate}\item Combination of multiset \end{enumerate}\end{enumerate}\end{frame}\section{}\begin{frame}\frametitle{Four Basic Counting Principles} There are {\it \bf FOUR} basics counting principle .they are,{\vspace{.5cm}} \begin{enumerate}\item {\Large \bf Addition} {\vspace{.5cm}}\item {\Large \bf Multiplication}{\vspace{.5cm}}\item {\Large \bf Subtraction}{\vspace{.5cm}}\item {\Large \bf Division }\end{enumerate} \end{frame}\section{}\begin{frame}\frametitle{Four Basic Counting Principles}{ \Large {\bf ADDITION PRINCIPLE :}} { \it{ Suppose that a set S is partitioned into parts S_1,S_2,\dots ,S_m. The number of of objects in S can be determined by finding the number of objects in each parts, and adding the numbers so obtaine:}} $$|S| = |S_1|+|S_2|+\dots +|S_m|$$ \end{frame}\section{}\begin{frame}\frametitle{Four Basic Counting Principles}{ \Large {\bf ADDITION PRINCIPLE :}} { \it{ Suppose that a set S is partitioned into parts S_1,S_2,\dots ,S_m. The number of of objects in S can be determined by finding the number of objects in each parts, and adding the numbers so obtaine:}} $$|S| = |S_1|+|S_2|+\dots +|S_m|$$ { {\bf Example : }A student wishes to take either a mathematics course or a biology course, but not both. If there are 4 mathematics courses and 3 biology courses for which the student has the necessary prerequisites, then the student can choose a course to take in 4+3=7 ways.}\\ \end{frame} \section{} \begin{frame}\frametitle{Four Basic Counting Principles} {\Large \bf MULTIPLICATION PRINCIPLE :} {\it Let S be a set of ordered pair $ (a,b) $ of objects, where the first object a comes from a set of size p, and for each choice of a there are q choices for object b. Then the size of S is $ p\times q:$}\\$$|S|= p \times q.$$ \end{frame} \section{} \begin{frame}\frametitle{Four Basic Counting Principles} {\Large \bf MULTIPLICATION PRINCIPLE :} {\it Let S be a set of ordered pair $ (a,b) $ of objects, where the first object a comes from a set of size p, and for each choice of a there are q choices for object b. Then the size of S is $ p\times q:$}\\$$|S|= p \times q.$$ The multiplication principle is actually a consequence of the addition principle. Let $a_1,a_2,\dots,a_p$ be the p different choices for the object a. We partition S into parts $S_1,S_2 ,\dots,S_p$ where S_i is the set of ordered pairs in S with first object a_i, (i=1,2,\dots,p). The size of each $|S_i|$ is q; hence, by the addition principle, \begin{center} $|S|=|S_1|+|S_2|+\dots+|S_p|$\\$=q+q+\dots+q$\\ $ =p\times q$ \end{center} \end{frame}\begin{frame}\frametitle{Four Basic Counting Principles} {\bf Example 1: }A student is to take two courses. The first meets at any one of 3 hours in the morning, and the second at any one of 4 hours in the afternoon. The number of schedules that are possible for the student is $ 3 \times 4 = 12 $.\\ \end{frame} \begin{frame}\frametitle{Four Basic Counting Principles } {\bf Example 1: }A student is to take two courses. The first meets at any one of 3 hours in the morning, and the second at any one of 4 hours in the afternoon. The number of schedules that are possible for the student is $ 3 \times 4 = 12 $.\\ {\bf Example 2: }Chalk comes in three different length, 8 different colors, and 4 different diameters. How many different kind of chalk are there ?\\{\bf Ans. }To determine a piece of chalk we carry out 3 different task : choose a length, choose a color, choose a diameter. By the multiplication principle, there are $ 3\times8\times4 = 96 $ different kinds of chalk.\end{frame} \section{} \begin{frame}\frametitle{Four Basic Counting Principles}{\Large \bf SUBTRACTION PRINCIPLE : }{\it Let A be a set and let U be a larger set containing $ A$. Let $$\overline{A}= \{ x\in U: x ot\in A\} $$be the complement of $ A$ in U. Then the number $|A|$ of objects in A is given by the rule $$|A|=|U|-|\overline{A}|.$$}\end{frame} \section{} \begin{frame}\frametitle{Four Basic Counting Principles}{\Large \bf SUBTRACTION PRINCIPLE : }{\it Let A be a set and let U be a larger set containing $ A$. Let $$\overline{A}= \{ x\in U: x ot\in A\} $$be the complement of $ A$ in U. Then the number $|A|$ of objects in A is given by the rule $$|A|=|U|-|\overline{A}|.$$}In applying the subtraction principle, the set U is usualy some natural set consisting of all the objects under discussion (the so-called universal set). Using the subtraction principle makes sense only if it is esier to count the number of objects in U and $|\overline{A}|$ then to count the number of objects in A.\end{frame} \section{} \begin{frame}\frametitle{Four Basic Counting Principles}{\bf Example 1: }Count the number of integers between 1 and 600, which are not divisible by 6.\\{\bf Ans . } Here U = the whole set = 600 \\ A = \{$x:x$ is between 1 and 600 and not divisible by 6 \} \begin{eqnarray*} \overline{A} &=& \{x : x ot\in A \} \\ &=& \{ x : x/6 \}\\ &=&600/6\\ &=& 100\\ therefore,\\ |A| &=& |U|-\overline{|A|} \\ &=&600-100\\ &=&500\\ \end{eqnarray*}\end{frame} \section{} \begin{frame}\frametitle{Four Basic Counting Principles}{\Large \bf DIVISON PRINCIPLE :}{\it Let S be a finite set that is partitioned into k parts in such a way that each part contains the same number of objects. Then the number of parts in the partition is given by the rule \\ $$k = \frac{|S|}{ number \hspace{.2cm} of \hspace{.2cm}objects\hspace{.2cm} in\hspace{.2cm} a\hspace{.2cm} part }$$}\end{frame} \section{} \begin{frame}\frametitle{Four Basic Counting Principles}{\Large \bf DIVISON PRINCIPLE :} {\it Let S be a finite set that is partitioned into k parts in such a way that each part contains the same number of objects. Then the number of parts in the partition is given by the rule \\ $$k = \frac{|S|}{ number \hspace{.2cm} of \hspace{.2cm}objects\hspace{.2cm} in\hspace{.2cm} a\hspace{.2cm} part }$$} Thus, we can determine the number of parts if we know the number of objects in S and the common value of the number of objects in the parts.{\vspace{1cm}} {\bf Example : } There are 740 pigeons in a collection of pigeonholes. If each pigeonhole contains 5 pigeons, the number of pigeonholes equals\\ $$740/5 =148$$\end{frame}\section{} \begin{frame}\frametitle{Permutations}Let r be a positive integer. By an r-$permutation$ of a set S of $ n$ elements, we understand an ordered arrangement of $ r$ of the n elements.\end{frame}\section{} \begin{frame}\frametitle{Permutations}Let r be a positive integer. By an r-$permutation$ of a set S of $ n$ elements, we understand an ordered arrangement of $ r$ of the n elements. If $$S = \{ a , b ,c \} $$ then the 1-$permutation$ of S are $$a \hspace{2cm} b \hspace{2cm} c$$\end{frame}\section{} \begin{frame}\frametitle{Permutations}Let r be a positive integer. By an r-$permutation$ of a set S of $ n$ elements, we understand an ordered arrangement of $ r$ of the n elements. If $$S = \{ a , b ,c \} $$ then the 1-$permutation$ of S are $$a \hspace{2cm} b \hspace{2cm} c$$\\the six 2-$permutations$ of S are $$ab \hspace{1cm} ac \hspace{1cm} ba \hspace{1cm} bc \hspace{1cm}ca \hspace{1cm} cb $$ \end{frame}\section{} \begin{frame}\frametitle{Permutations}Let r be a positive integer. By an r-$permutation$ of a set S of $ n$ elements, we understand an ordered arrangement of $ r$ of the n elements. If $$S = \{ a , b ,c \} $$ then the 1-$permutation$ of S are $$a \hspace{2cm} b \hspace{2cm} c$$\\the six 3-$permutations$ of S are\\ $$abc \hspace{1cm}acb \hspace{1cm} bac \hspace{1cm} bca \hspace{1cm} cab \hspace{1cm} cba$$ \end{frame}\section{} \begin{frame}\frametitle{Permutations}{\bf Theorem 1 : }For n and r positive integers with $ r \le n$.\\ $$P(n , r) = n \times (n-1) \times .....\times (n - r + 1)$$ \end{frame}\section{} \begin{frame}\frametitle{Permutations}{\bf Theorem 1 : }For n and r positive integers with $ r \le n$.\\ $$P(n , r) = n \times (n-1) \times .....\times (n - r + 1)$$ {\bf Results :} For a non-negative integer n, we define n! by $$n!=n\times(n-1)\times\dots\times2\times1$$ with the convention that 0!=1. We may then write $$P(n,r)=\frac{n!}{(n-r)!}$$ \end{frame}\section{} \begin{frame}\frametitle{Permutations}{\bf Theorem 1 : }For n and r positive integers with $ r \le n$.\\ $$P(n , r) = n \times (n-1) \times .....\times (n - r + 1)$$ {\bf Results :} For a non-negative integer n, we define n! by $$n!=n\times(n-1)\times\dots\times2\times1$$ with the convention that 0!=1. We may then write $$P(n,r)=\frac{n!}{(n-r)!}$$ {\bf Example 1 : } The number of 4-letter ``words" that can be formed by using each of letters a, b, c, d, e at most once is $P(5,4)$, and this equals 5!/ (5-4)! = 120. The number of 5-letter words equals $P(5,5)$, which is also 120 .\end{frame}\section{} \begin{frame}\frametitle{Permutations}{\bf Theorem 2: } The number of circular r-permutations of a set of n elements is given by $$\frac{P(n,r)}{r} = \frac{n!}{r\times(n-r)!}$$In particular, the number of circular permutations of n elements is $(n-1)!$\end{frame}\section{} \begin{frame}\frametitle{Permutation}{\bf Theorem 2: } The number of circular r-permutations of a set of n elements is given by $$\frac{P(n,r)}{r} = \frac{n!}{r\times(n-r)!}$$In particular, the number of circular permutations of n elements is $(n-1)!$\end{frame}\section{} \begin{frame}\frametitle{Permutations}There are some results in permutation. they are {\vspace{.5cm}} \begin{enumerate} \item $P(n,n)=n!$\{\vspace{.3cm}} \item $P(n,0)=\frac{n!}{(n-0)!}=1$\{\vspace{.3cm}} \item $P(n,1)=1$\{\vspace{.3cm}} \item $P(n,n-1)=n!$\end{enumerate}\end{frame}\section{} \begin{frame}\frametitle{Permutation of Multiset}If S is a multiset, an r-$permutation$ of S is an ordered arrangement of r of the objects of S. If the total number of objects of S is n (counting repetitions), then an n-permutation of S will also be called a $permutation$ of S. For example, if $S= \{2.a, 1.b, 3.c\}$, then $$acbc \hspace{1cm} cbcc$$ are 4-permutations of S, while $$abccca$$is a permutation of S.\\{\bf Remark :} The multiset S has no 7-permutations since 7 is greater then 2+1+3=6, the number of objects of S.\end{frame}\section{} \begin{frame}\frametitle{Permutation of Multiset}If S is a multiset, an r-$permutation$ of S is an ordered arrangement of r of the objects of S. If the total number of objects of S is n (counting repetitions), then an n-permutation of S will also be called a $permutation$ of S. 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Then the number of r-permutations of S is k^r.\end{frame}\section{} \begin{frame}\frametitle{Permutation of Multiset}{\bf Theorem 1:} Let S be a multiset with objects of k different types, where each has an infinite repetition number. Then the number of r-permutations of S is k^r.\\{\vspace{1cm}}{\bf Example 1:} What is the number of ternary numerals with at most 4 digits.\\{\bf Ans. } The answer to this question is the number of 4-permutations of the multiset $\{ \infty.0, \infty.1, \infty.2 \}$ or of the multiset $ \{ 4.0, 4.1, 4.2 \} $. By previous Theorem, this number equals $3^4=81$.\end{frame}\section{} \begin{frame}\frametitle{Permutation of Multiset}{\bf Theorem 2: }Let S be a multiset with objects of k different types with finite repetition numbers $n_1,n_2,\dots,n_k$, respectively. Let the size of S be $n=n_1+n_2+\dots+n_k$. Then the number of permutations of S equals $$ \frac{n!}{n_1!n_2!\dots n_k!}.$$ \end{frame}\section{} \begin{frame}\frametitle{Permutation of Multiset}{\bf Theorem 2: }Let S be a multiset with objects of k different types with finite repetition numbers $n_1,n_2,\dots,n_k$, respectively. Let the size of S be $n=n_1+n_2+\dots+n_k$. Then the number of permutations of S equals $$ \frac{n!}{n_1!n_2!\dots n_k!}.$$ {\bf Example 2: } The number of permutation of the letters in the word MISSISSIPPI is $$\frac{11!}{1!4!4!2!},$$ since this number equals the number of permutations of the multiset $\{ 1.M, 4.I, 4.S, 2.P \}$\end{frame}\section{} \begin{frame}\frametitle{Combinations}Let r be a non negative integer. By an r-$combination$ of a set S of n elements, we understand an unordered selection of r of the n objects of S.\end{frame}\section{} \begin{frame}\frametitle{Combinations}Let r be a non negative integer. By an r-$combination$ of a set S of n elements, we understand an unordered selection of r of the n objects of S. If $S= \{a,b,c,d\}$, then$$ \{a,b,c\} , \{a,b,d\} ,\{a,c,d\}, \{b,c,d\}$$are the four 3-$cobination$ of S.We denote by $\binom{n}{r}$ the number of r-combinations of an n-element set.\end{frame}\section{} \begin{frame}\frametitle{Combinations}Let r be a non negative integer. By an r-$combination$ of a set S of n elements, we understand an unordered selection of r of the n objects of S. If $S= \{a,b,c,d\}$, then$$ \{a,b,c\} , \{a,b,d\} ,\{a,c,d\}, \{b,c,d\}$$are the four 3-$cobination$ of S.We denote by $\binom{n}{r}$ the number of r-combinations of an n-element set. Obviously, \\ $$\binom{n}{r} = 0 \hspace{1cm} if \hspace{.5cm} r>n.$$ also, $$\binom{0}{r}=0 \hspace{1cm} if \hspace{.5cm} r>0.$$\end{frame}\section{} \begin{frame}\frametitle{Combinations} the following additional facts are readily seen to be true for each non-negative integer n:{\vspace{.8cm}} \begin{enumerate} \item $\binom{n}{0} = 1$\{\vspace{.4cm}} \item $\binom{n}{1} = n$\{\vspace{.4cm}} \item $\binom{n}{n} = 1 $\{\vspace{.4cm}} \item $\binom{0}{0} = 1$\{\vspace{.4cm}} \end{enumerate}\end{frame}\section{} \begin{frame}\frametitle{Combinations}{\bf Theorem :} For $ 0 \le r \le n,$$$P(n,r)=r!{\binom{n}{r}}.$$ Hence, $$\binom{n}{r} = \frac{n!}{r!(n-r)!}.$$\pause{\bf Example :} Twenty-five points are chosen in the plane so that no three of them are collinear. How many straight lines do they determine?\\ {\bf Ans.} Since no three of the points lie on a line, every pair of points determines a unique straight line. Thus, the number of straight lines determined equals the number of 2-combinations of a 25-element set, and this is given by $$\binom{25}{2}=\frac{25!}{2!23!}=300.$$\end{frame}\section{} \begin{frame}\frametitle{Combinations of Multiset}If S is a multiset, then an r-$combination$ of S is an unordered selection of r of the objects of S. Thus, an r-combination of S is itself a multiset, a submultiset of S.\begin{enumerate}\item If S has n objects, then there is is only one n-combination of S, namely, S itself.\item If S contains objects of k different types, then there are k 1-combinations of S.\end{enumerate}\end{frame}\section{} \begin{frame}\frametitle{Combinations of Multiset}If S is a multiset, then an r-$combination$ of S is an unordered selection of r of the objects of S. Thus, an r-combination of S is itself a multiset, a submultiset of S.\begin{enumerate}\item If S has n objects, then there is is only one n-combination of S, namely, S itself.\item If S contains objects of k different types, then there are k 1-combinations of S.\end{enumerate}\end{frame}\section{} \begin{frame}\frametitle{Combinations of Multiset}{\bf Theorem :} Let S be a multiset with objects of k types, each with an infinite repetition number. Then the number of r-combinations of S equals$$\binom{r+k-1}{r}=\binom{r+k-1}{k-1}$$\end{frame}\section{} \begin{frame}\frametitle{Combinations of Multiset}{\bf Theorem :} Let S be a multiset with objects of k types, each with an infinite repetition number. Then the number of r-combinations of S equals$$\binom{r+k-1}{r}=\binom{r+k-1}{k-1}$$\end{frame}\section{} \begin{frame}\frametitle{Combinations of Multiset}{\bf Example :} A bakery boasts 8 varieties of doughnuts. If a box of doughnuts contain 1 dozen, how many different options are there for a box of doughnuts? \\{\bf Ans. } It is assumed that the bakery has on hand a large number (at least 12) of each variety. This is a combination problem, since we assume the of order of the doughnuts in a box is a box is irrelevant for the purchaser's purpose. The number of different options for boxes equals the number of 12-combinations of a multiset with objects of 8 types, each having an infinite repetition number. This number equals \\ $$\binom{12+8-1}{12}=\binom{19}{12}$$\end{frame}\section{} \begin{frame}\frametitle{Conclusion}We have discuss these following topics :{\vspace{.2cm}}\begin{enumerate} \item Using the Counting